EXPERIMENTAL WIRELESS
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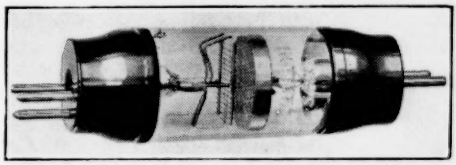
Editorial.

The National Radio Exhibition, Olympia, 1927.

IN our October issue we promised to deal individually with some of the outstanding features of the Exhibition. There was general agreement that it was a very successful exhibition from every point of view, although, as on previous occasions, one felt the unavoidable limitations due to the lack of any facilities for demonstrating loud-speakers in operation. This is very much to be regretted at the present time when so many people are ignorant of the advances which have been made in loud-speaker design, and still fancy that good quality can only be obtained on the headphones. Many exhibitors got over the difficulty to some extent by inviting visitors to show-rooms in the neighbourhood of Olympia where their loud-speakers and sets could be seen and heard in operation. It is certain, however, that the present arrangement is preferable to any half-hearted attempt to give demonstrations within the building; if done at all it would have to be done very well. After all, the radio exhibitor is in no worse position than the motor exhibitor, whose optimistic pronouncements as to speed and petrol consumption have to be tested elsewhere than in the Show.

The exhibit can undoubtedly be taken as a fair guide to the present tendencies in broadcast receiver design and operation. From this point of view one can safely say that crystal detectors and headphones are almost obsolete so far as the industry is concerned, that except in portable sets, the horn type of loud-speaker has been displaced by the large diaphragm type for domestic use; that a battle is being waged between the dry battery, the accumulator, and the mains supply unit as a source of anode current; that makers are realising the importance of metal linings and screens in the radio-frequency sections of cabinets, and that a feverish interest is being shown in new types of valves, especially those of the screened or shielded anode type with which we dealt in our last issue.



The Marconi and Osram type of screened anode valve.

With one or two exceptions the number of adjusting knobs on the front of the receiver has been reduced to a minimum. This is partly due to improvements in valve characteristics making it no longer necessary to adjust the filament currents individually or even at all; it is partly due to the development of the unified or "gang" control of two or more condensers, making it possible to tune all the high frequency circuits

simultaneously. This is such an important advantage to the ordinary user that one is justified in sacrificing some amplification to obtain such simplicity in manipulation.

With reference to the question of anode current supply there is the same choice between the troublesome technical ideal as represented by the absolutely steady and low resistance wet battery, either primary or secondary, the dry battery which in its larger sizes is almost ideal and free from trouble beyond the necessity of occasional renewal, and the mains unit which can be made to approach as near to the ideal as one is prepared to pay for in the way of reduction of resistance and increase of smoothing apparatus.

The B.T.H. screened valve.

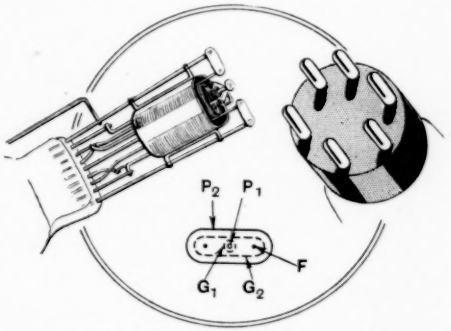
Mention should be made of the great number of portable sets exhibited, some of them of beautiful workmanship both inside and out; but one would be more inclined to discuss them if they had been able to show the quality of their reproduction. Wonderful ingenuity had been shown in arranging a complete superheterodyne set together with all the batteries, frame aerials and loud-speaker within the compass of a small suit case.

The high standard of quality which was the outstanding general feature both of the sets and of the components was also noticeable in the ammeters and voltmeters which are now put forward for wireless work. The Ferranti Company showed a number of single and multi-range instruments mounted in moulded cases of insulating material of either the portable or flush pattern. These varied in price from 30s. to 55s., and would make a strong appeal to the serious experimenter.

We do not propose, however, to attempt any complete enumeration of the exhibits, as this has been done very fully by our contemporary, the *Wireless World*.* We must, however, describe in some detail two novel forms of valve each of which attracted a large amount of attention.

The Midgley-Ediswan One-Valve Loud-Speaker Set.

An interesting and important novelty was shown by the Ediswan Company. This was a one-valve set designed by Mr. A. H. Midgley which, it is claimed, will give excellent headphone reception of many foreign stations, loud-speaker reception up to 20 miles from a main station and up to 80 miles from Daventry. The valve, however, is not an ordinary one but a six-pin bulb containing two anodes and two grids; that is, the bulb really contains two valves with a common filament. Its performance should, therefore, be compared with that of a two- or three-valve set seeing that the retail price of the new valve is 22s. 6d. The single filament will take little current, but with the new three-electrode valves taking only 0.075 ampere, this is not such an important point as the claim that the Midgley valve has an H.T. current consumption of about a third of that ordinarily necessary for equivalent loud-speaker results.



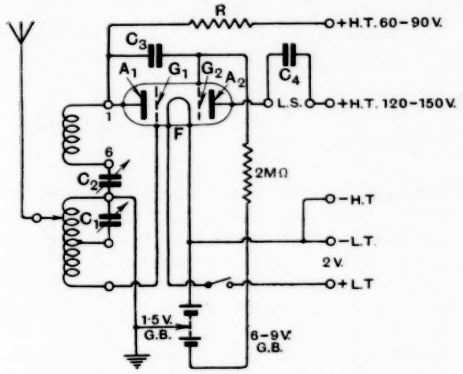
Details of the Midgley-Ediswan valve.

The electrode arrangement is cylindrical, the outer cylinder forming one anode and a central rod the other; between them are the two cylindrical grids with the filament between them. The diagram of connections is reproduced.

The action appears to be somewhat as follows: The aerial-earth circuit is tuned

* *Wireless World*, 28th September and 5th October.

by means of the condenser C_1 and the voltage across C_1 is stepped up and applied to the grid G_1. The valve G_1A_1 probably acts as an anode-bend rectifier, the high-frequency component of the anode current passing through the reaction coil 1—6 and



Circuit of the Ediswan receiver.

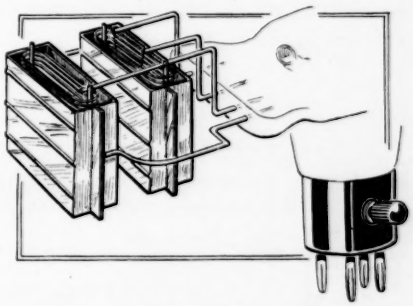
the condenser C_2, while the audio-frequency component passing through the resistance R, affects the potential of G_2 through the condenser C_3. In other words, the valve G_1A_1 is resistance-capacity coupled to the valve G_2A_2. The condenser C_4 shown in parallel with the loud-speaker is probably necessary to bypass the radio-frequency currents due to the effect of the radio grid G_1 on the emission from the filament to A_2.

It is claimed that the set does not re-radiate, presumably owing to the coupling being fixed at a point below the oscillation limit.

It will be interesting to compare the performance of the set with that of two ordinary valves in the same circuit; this will be an easy matter as the Ediswan Co. propose to supply the components to enable amateurs to build Midgley sets.

The Robinson-R.I. and Varley Interdyne.

Considerable interest was aroused by a set exhibited by the R.I.-Varley Co., in which a new type of valve was employed. This valve, to which the name Interdyne has been given, is due to Dr. J. Robinson. In the ordinary valve the capacity between the anode and grid necessitates the use of an external neutralising condenser to counterbalance the reaction and consequent tendency to oscillation. Dr. Robinson conceived the idea of placing the neutralising condenser inside the bulb of the valve and, moreover, of making it a replica of the anode-grid element the capacity of which it is intended to counterbalance. In addition to the working anode and grid, the interdyne valve has thus a dummy anode and grid. It is claimed that better results are thereby obtained. It would be interesting to try to utilise burnt out valves as neutralising condensers to valves of the same type. One drawback



Arrangement of the Robinson-R.I. and Varley valve.

in each case as compared with the usual type of neutralising condenser is the impossibility of adjustment if the neutralisation is not exact; it is quite possible, however, that the absence of adjustment may be regarded as one of the main advantages of the new device.

Some Notes on the Effect of Coupling between Loop and Beating Oscillator Circuits in a Superheterodyne Receiver.

By E. H. Ullrich, M.A., A.M.I.E.E., and
A. H. Reeves, A.C.G.I., D.I.C.
(International Standard Electric Corporation.)

WHEN the beating oscillator and loop circuits of a superheterodyne are coupled to the grid of the first detector valve in any of the more common ways, there usually exists a certain degree of coupling between the beating oscillator and loop circuits themselves. Fig. 1 gives a circuit diagram in which the beating oscillator voltage is introduced inductively. The direct consequence of this coupling is

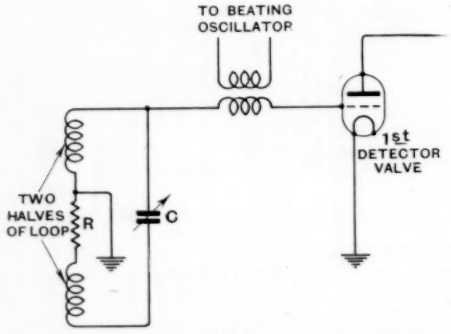


Fig. 1.

that the tuning of either circuit affects the other, the degree of inconvenience caused depending on the frequency to be received and the frequency of the intermediate amplification. In the ordinary broadcasting range the trouble is not a serious one, but at wavelengths less than 100 metres it becomes very difficult to tune in a signal properly, unless the intermediate frequency is raised considerably above 100 kilocycles with consequent loss of amplification, selectivity and stability. If the tuning process followed is that of varying the oscillator condenser setting until the signals in the headphones are of maximum loudness and then varying the loop tuning until a further maximum is obtained, it is found that the same station may be tuned in at different times on widely different settings of both loop and oscillator condensers. Furthermore, with values of coupling usual at less than 100 metres the frequency of the beating oscillator is a discontinuous function of both tuning condensers; in other words, as the latter is increased, there will be a point at which an infinitesimally small change of capacity causes a finite frequency change. In addition there is the hysteresis effect, well known in connection with coupled oscillator circuits. The trouble may be mitigated by reducing the coupling of the beating oscillator to the grid circuit of the detector, but this will lower the beating oscillator voltage on the detector and thereby cut down the signal in the headphones; the advantage gained hardly warrants the sacrifice in amplification.

The sudden change of frequency, which may be caused by variation of tuning of a secondary circuit, has been investigated by several people.* The writers, however, do not know of any published discussion of the precise circuit involved here and feel that a short mathematical investigation may be of interest.

Let us consider Fig. 2, in which the beating oscillator voltage is introduced on to the grid of the detector by means of a small coil of inductance L_2 and mutual inductance M_2 with the inductance in the oscillatory circuit of the oscillator. For simplicity we shall treat the impedance of L_2 as negligible

* Rogowski. *Die Frequenzsprünge des Zwischenkreisröhrensenders Arch. f. Elektrot.* 10, 1 (1921), etc.

compared with the impedance at beating oscillator frequency of the loop circuit $L_3\ R_3\ C_3$.

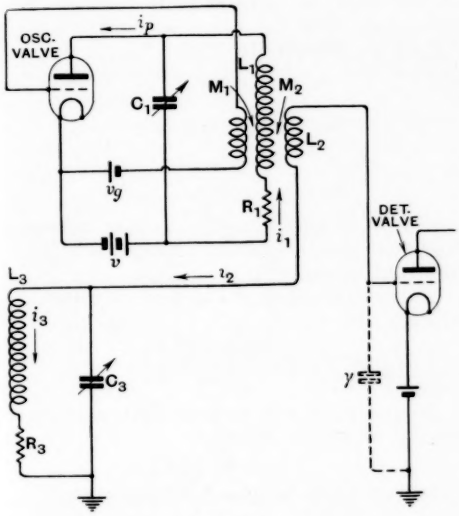


Fig. 2.

Attaching the usual meanings to the symbols used, we have the equations :—

$$e_p = V - \int \frac{(i_p - i_1)dt}{C_1} \quad \ldots \quad (1)$$

$$\int \frac{i_p - i_1)\,dt}{C_1} = R_1 i_1 + L_1 \frac{di_1}{dt} - M_2 \frac{di_2}{dt} \quad (2)$$

$$R_p i_p = e_p + \mu e_g = e_p - \mu\left(v_g + M_1 \frac{di_1}{dt}\right) \quad (3)$$

whence we get

$$R_p\left[i_1 + C_1 R_1 \frac{di_1}{dt} + C_1 L_1 \frac{d^2 i_1}{dt^2} - C_1 M_2 \frac{d^2 i_2}{dt^2}\right]$$
$$= V - \mu v_g - \left(R_1 i_1 + L_1 \frac{di_1}{dt} - M_2 \frac{di_2}{dt}\right) - \mu M_1 \frac{di_1}{dt} \quad (4)$$

We have also

$$M_2 \frac{di_1}{dt} = L_3 \frac{di_3}{dt} + i_3 R_3 + \int \frac{i_3 dt}{\gamma} \quad (5)$$

$$L_3 \frac{di_3}{dt} + R_3 i_3 = \int \frac{(i_2 - i_3)\,dt}{C_3} \quad \ldots \quad (6)$$

whence we get the equation—

$$-R_p C_1 M_2^2 C_3 L_3 \frac{d^6 i_3}{dt^6}$$
$$-(R_p C_1 M_2^2 C_3 R_3 + M_2^2 C_3 L_3)\frac{d^5 i_3}{dt^5}$$
$$+\left\{-M^2{}_2 C_3 R_3 - M_2^2 R_p C_1 + C_1 L_1 R_p \left(L_3 + \frac{C_3 L_3}{\gamma}\right)\right\}\frac{d^4 i_3}{dt^4}$$
$$+\left\{-M_2^2 + C_1 L_1 R_p\left(R_3 + \frac{C_3 R_3}{\gamma}\right) + \left(L_3 + \frac{C_3 L_3}{\gamma}\right)\left(C_1 R_1 R_p + L_1 + \mu M_1\right)\right\}\frac{d^3 i_3}{dt^3}$$
$$+\left\{\frac{C_1 L_1 R_p}{\gamma} + \left(R_3 + \frac{C_3 R_3}{\gamma}\right)(C_1 R_1 R_p + L_1 + \mu M_1) + \left(L_3 + \frac{C_3 L_3}{\gamma}\right)(R_1 + R_p)\right\}\frac{d^2 i_3}{dt^2}$$
$$+\left\{\frac{1}{\gamma}(C_1 R_1 R_p + L_1 + \mu M_1) + (R_p + R_1)\left(R_3 + \frac{C_3 R_3}{\gamma}\right)\right\}\frac{di_3}{dt}$$
$$+\frac{(R_p + R_1)}{\gamma} i_3 = 0 \quad \ldots \quad (7)$$

The solutions of this equation are given by an algebraic equation of the sixth degree, which may be rearranged as follows :—

$$\left[x^2 + x\left\{\frac{R_1}{L_1} + \frac{(L_1 + \mu M_1)}{C_1 R_p L_1}\right\} + \frac{(R_p + R_1)}{C_1 R_p L_1}\right]\left[x^2 + \frac{R_3}{L_3}x + \frac{1}{L_3(C_3 + \gamma)}\right]$$
$$-\frac{M_2^2 C_3}{L_1\left(1 + \frac{C_3}{\gamma}\right)}\left[x^6 + \left(\frac{R_3}{L_3} + \frac{1}{R_p C_1}\right)x^5 + \left(\frac{R_3}{L_3}\cdot\frac{1}{R_p C_1} + \frac{1}{C_3 L_3}\right)x^4 + \frac{x^3}{R_p C_1 C_3 L_3}\right] = 0 \quad (8)$$

In the case where M_2 is zero the solution of equation (7) is very closely—

$$i_3 = Ae^{-\frac{1}{2}\left\{\frac{R_1}{L_1} + \frac{(L_1 + \mu M_1)}{C_1 R_1 L_1}\right\}t} \sin\left(\frac{t}{\sqrt{L_1 C_1}} + \epsilon_1\right) + Be^{-\frac{R_3 t}{2L_3}} \sin\left(\frac{t}{\sqrt{L_3(C_3 + \gamma)}} + \epsilon_2\right) \quad (9)$$

where A, B, ϵ_1 and ϵ_2 are arbitrary constants.

The frequencies correspond to the natural frequencies of the simple loop and oscillator circuits, and the oscillation due to the loop circuit dies away at once. The oscillation due to the beating oscillator circuit will build up, if M_1 is negative and sufficiently large. When, however, the strength of oscillation is greater than can be handled

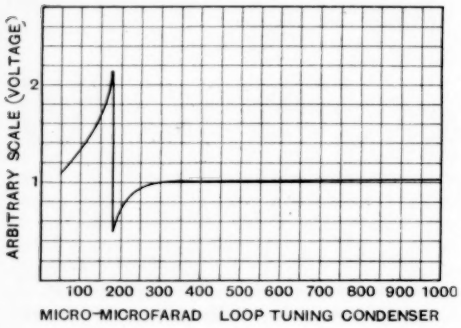


Fig. 3.

by the straight part of the valve characteristic, R_p and μ change to new average values such that

$$\frac{R_1}{L_1} + \frac{(L_1 + \mu M_1)}{C_1 R_p L_1} = 0$$

The effect of the presence of a small M_2 is to change slightly the two theoretical frequencies of oscillation and to change their decrements. We may write in this case

$$i_3 = Ae^{-\alpha_1 t} \sin(\omega_1' t + \varepsilon_1) + Be^{-\alpha_3 t} \sin \omega_3' t + \varepsilon_3) \quad (10)$$

where ω_1' is approximately

$$\frac{1}{\sqrt{L_1 C_1}}$$

and ω_3' is approximately

$$\frac{1}{\sqrt{L_3(C_3 + \gamma)}}$$

The decrement of the oscillation corresponding to the loop circuit is approximately

$$\alpha_3 = \frac{R_3}{2L_3} - \frac{M_2^2 \gamma^2 \omega_3^3 \delta_3}{2L_1(C_3 + \gamma)(4\varepsilon^2 + \delta_3^2)} \quad (11)$$

where

$$\omega_1 \equiv \frac{1}{\sqrt{L_1 C_1}}$$

$$\omega_3 \equiv \frac{1}{\sqrt{L_3(C_3 + \gamma)}}$$

$$\omega_3 \equiv \omega_1(1 - \varepsilon)$$

and $$\delta_3 \equiv R_3/\omega_3 L_3$$

If $\varepsilon > \delta_3$, this is approximately

$$\alpha_3 = \frac{\omega_3 \delta_3}{2} - \frac{M_2^2 \gamma^2 \omega_3^5 \delta_3}{8\varepsilon^2} \times \frac{L_3}{L_1} \quad \ldots (12)$$

If ε is reduced until α_3 just becomes negative, the frequency changes suddenly from ω_1' to ω_3'.

The critical value of ε is given by $\alpha_3 = 0$; whence

$$\varepsilon_c = \pm \frac{M_2 \gamma \omega_3^2}{2} \sqrt{\frac{L_3}{L_1}} \quad \ldots (13)$$

From (11) it will be seen that α_3 cannot become negative unless

$$M_2 > \frac{\delta_3}{\gamma \omega_3^2} \sqrt{\frac{L_1}{L_3}}$$

Let us consider the effect on α_3 of raising the frequency limits of the system, *e.g.*, changing from a set for broadcasting wavelengths to one operating between 10 and 100 metres.

If the oscillator is oscillating powerfully the voltage on the plate will fall to a low

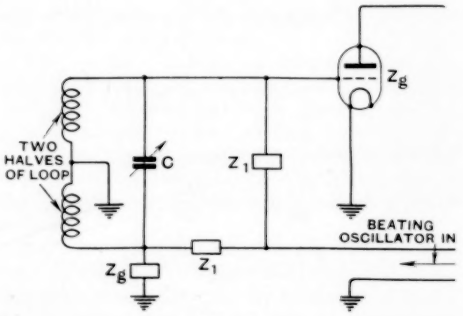


Fig. 4.

value, *i.e.*, the voltage across $L_1 R_1$ will depend chiefly on the plate battery. Now the volts picked up in L_2 will depend chiefly on the requirements of the detector tube, *i.e.*, will be practically independent of

frequency; and the ratio of this voltage to the voltage across L_1R_1 is very nearly M_2/L_1. Actually, however, the tendency is for oscillators to oscillate less powerfully at the shorter wavelengths, so that M_2/L_1 tends to increase somewhat with frequency; *i.e.*, $M_2C_1\omega_1^2$ tends to increase with frequency. γ is constant and C_1 and C_3 are

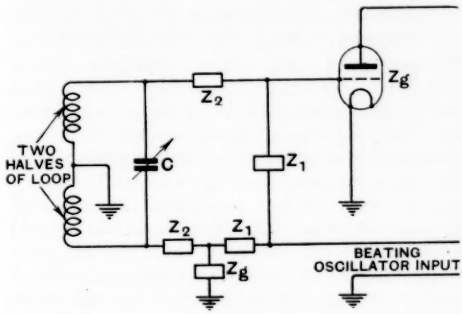


Fig. 5.

variable condensers, which cannot be conveniently reduced much below 100$\mu\mu$F maximum capacity, whatever the frequency limits of the receiver. Therefore $M_2\omega_1^2$ and also $M_2\omega_3^2$ tend to increase with frequency. L_1 and L_3 must be chosen so that they can be made to resonate with C_1 and C_3 respectively at the lower frequency limit. They will naturally be approximately equal. Therefore from (13) ϵ_c tends somewhat to increase as the frequency limits of the system are raised.

In consequence of this it follows that, if the frequency limits of a superheterodyne receiver are multiplied by a factor "p" the difficulties due to the coupling between loop and beating oscillator circuits will be increased, unless either the intermediate frequency is multiplied by a factor somewhat greater than "p," or precautions are taken to prevent the tuning of the loop circuit from affecting the beating oscillator frequency.

In Fig. 4 a means of introducing the beating oscillator voltage on to the grid of the first detector is shown, wherein one point, at high beating oscillator potential to earth, is coupled through two equal impedances Z_1 to the extremities of the loop tuning condenser C. From the symmetry of the system it will be seen that no beating oscillator voltage will exist across condenser C, so that the tuning of the loop cannot affect the beating oscillator circuit. In other words, the coupling between the two circuits is nil. It will be seen, however, that in this case the currents flowing in the two halves of the loop are equal and opposite, and consequently, the inductance of each half of the loop will be practically zero to the beating oscillator current. In other words, from the point of view of the beating oscillator, the grid of the first detector is practically short-circuited to earth, so that no beating oscillator voltage will be impressed upon the grid. It is, therefore, necessary to introduce two equal impedances Z_2, as shown in Fig. 5. Although the impedances Z_1 and Z_2 may be of any nature, they will introduce undesirable losses if they contain large resistive components. The circuit of Fig. 6 is, therefore, particularly applicable, the 100,000-ohm leak across condenser D being introduced to stabilise the grid bias, and condenser F having the same value as condensers D and E in series. The presence of the leak across condenser D and the omission of impedance Z_g do not cause any appreciable disturbance of balance of the system. The degree of independence

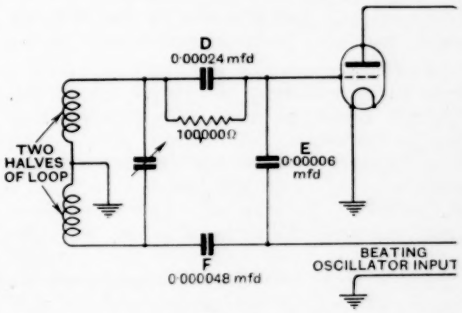


Fig. 6.

of the loop and beating oscillator circuits may be tested by noting the variation of the plate current of the first detector, when the beating oscillator is working and the loop tuning condenser is being slowly turned. When this experiment was carried out at 200 metres, no variation whatever of the plate current was observed, showing that a very

high degree of balance had been obtained. At 50 metres the grid-to-plate capacity of the tube had a very appreciable disturbing effect and had to be compensated for by a small condenser between plate and one side of the loop, as seen in Fig. 7, and the condenser corresponding to *F* in Fig. 6 was made variable to secure an accurate balance.

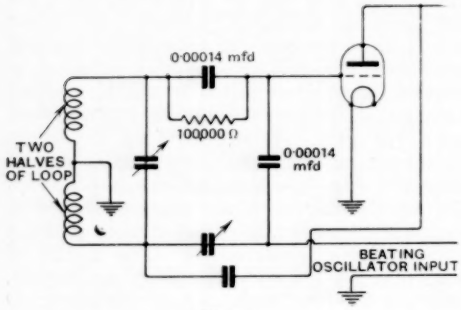


Fig. 7.

The variation of grid-filament volts at beating oscillator frequency, as indicated by the change of plate current as the loop circuit was varied through the point of resonance to the beating oscillator, was then easily reduced to less than $\pm\frac{1}{2}$ per cent. In Fig. 3 the beating oscillator voltage between grid and filament of the detector valve has been plotted against loop tuning condenser for the circuit of Fig. 1. We have assumed—

a beating oscillator frequency of 1,050,000 cycles ;

a loop inductance of 0.1265 millihenries (giving resonance with 200$\mu\mu$F) ;

a loop resistance of 20 ohms ; and

a grid-to-filament capacity of 30$\mu\mu$F (this including the capacity to earth of the coupling coil of negligible inductance).

The grid-filament beating oscillator voltage is seen to change by 300 per cent. as the loop tuning is varied. It must be remembered, also, that if the loop resistance had been 10 instead of 20 ohms, the maximum and minimum beating oscillator volts in Fig. 3 would have been doubled and halved respectively, so that the variation would have been 1,500 per cent.

The effect of the three condensers *D*, *E* and *F* in Fig. 6 is to increase the minimum capacity across the loop by 0.000024μF and to reduce the signal voltage that reaches the grid by 20 per cent. Without the device, however, the sensitivity is very considerably more than 20 per cent. below the maximum at 100 metres, as the beating oscillator coupling must be kept very low in order to avoid the disturbing effects mentioned above. The beating oscillator voltage on the detector grid is in consequence very small.

Theory of Receiving Aerials.

By F. M. Colebrook, B.Sc., D.I.C., A.C.G.I.

THE writer received some time ago a copy of a very interesting paper by Mr. E. B. Moullin, "On the Current Induced in a Wireless Telegraph Receiving Antenna." The following article is an extension of the above admirable introduction to a somewhat neglected subject. The writer wishes to make the fullest possible acknowledgments to Mr. Moullin, and does not claim to have done anything more than generalise the original work and to have emphasised certain conclusions which, though implicit in Mr. Moullin's work, were not specifically stated.

It so happened that the writer was actually engaged on the problem at the time he received the paper referred to, with the object of answering the following questions :—

1. What is the nature of the effective impedance of a receiving aerial from the point of view of associated receiving apparatus ?

2. Is the effective impedance dependent on (*a*) the nature of the tuning or receiving circuit ; (*b*) the distribution of the electric field due to the signal ?

3. What part is played by the distributed resistance of an aerial ?

4. What is the effective height of an aerial, and does it depend on (*a*) the tuning circuit conditions ; (*b*) the field distribution ?

5. Is there any optimum distribution for a given length of aerial ?

The problem had been tackled on rather different lines from Mr. Moullin's, and unsuccessfully, owing to a mishandling of the mathematics involved. The application of Mr. Moullin's method led to the solution of these questions, and as a result of the familiarity with the subject so gained the writer has since found that his original method, properly handled, is equally effective and leads to the same conclusions.

Analysis is apt to be dullish reading. The writer will therefore content himself with presenting the merest outline of the work, just sufficient to enable anyone sufficiently enthusiastic to check it for himself.

The physical conditions of the problem and the more important symbols are represented in Fig. 1. The current co-ordinates x_1 and x_2 follow the actual lines of the vertical and horizontal parts of the aerial structure. It is assumed that each part of the aerial has a certain resistance (including radiation resistance), inductance and capacity per unit length, these being uniform in each part. This may not be strictly true, as proximity to the ground may cause a local variation of capacity. This is not likely to be very pronounced, however, and will be referred to later.

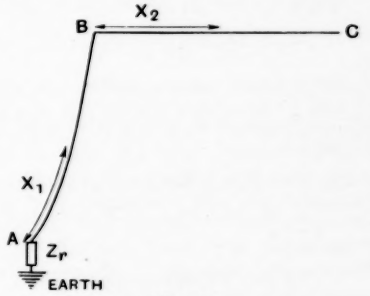


Fig. 1.

In most cases of distant reception the field due to the signal will probably be sensibly uniform over the space occupied by the aerial. The aerial itself, however, is not a geometrical design drawn on paper, but a number of wires hung up in space in some convenient way. In general, therefore, it will not be symmetrically disposed with regard to the field, and the E.M.F. induced per unit length may be far from uniform. This has been taken into account by representing the field intensities as $f_1(x_1)$ and $f_2(x_2)$ respectively for the vertical and horizontal parts of the aerial, *i.e.*, the E.M.F. induced in an element dx_1 will be $f_1(x_1)\,dx_1$. The top part of the aerial is shown as a single line, for the sake of simplicity. It may, of course, consist of more than one line but this does not affect the state of affairs provided the lines run parallel and are not widely separated compared with their length.

Considering the vertical part first, the differential equations for $\mathbf{v}_1$ and $\mathbf{i}_1$ will be

$$(R_1 + j\omega L_1)\mathbf{i}_1 - \mathbf{e}_1 = -\partial\mathbf{v}_1/\partial x_1 \quad \ldots \quad (1)$$

$$j\omega C_1 \mathbf{v}_1 = -\partial\mathbf{i}_1/\partial x_1 \quad \ldots \quad (2)$$

where $$\mathbf{e}_1 = f_1(x_1) \quad \ldots \quad (3)$$

Here $\mathbf{v}_1$ and $\mathbf{i}_1$ are vectors representing the potential with respect to earth and the current respectively, and $\omega/2\pi$ is the frequency of the signal E.M.F., assumed to be a pure continuous wave for the sake of simplicity. These equations are essentially the same as those used by Mr. Moullin, except that R_1 is introduced and $\mathbf{e}_1$ is not regarded as independent of x_1.

The differentiation of these equations with respect to x_1 gives two more, *i.e.*,

$$P_1^2\mathbf{i}_1 - j\omega C_1\mathbf{e}_1 = \partial^2\mathbf{i}_1/\partial x_1^2 \quad \ldots \quad (4)$$

$$P_1^2\mathbf{v}_1 + \partial\mathbf{e}_1/\partial x_1 = \partial^2\mathbf{v}_1/\partial x_1^2 \quad \ldots \quad (5)$$

where

$$P_1^2 = (R_1 + j\omega L_1)\, j\omega C_1 \quad \ldots \quad (6)$$

It will be found that the solutions of these equations are

$$\mathbf{i}_1 = \mathbf{A}_1 \sinh P_1 x_1 + \mathbf{B}_1 \cosh P_1 x_1 + \psi_1(x_1) \quad (7)$$

$$\mathbf{v}_1 = -Z_1(\mathbf{A}_1 \cosh P_1 x_1 + \mathbf{B}_1 \sinh P_1 x_1) - \phi_1(x_1) \quad (8)$$

where

$$\psi_1(x_1) = j\omega C_1 \frac{\mathbf{e}_1}{P_1^2 - D^2} \quad \ldots \quad (9)$$

(D being written for the operation $\partial/\partial x$)

$$\phi_1(x_1) = \frac{1}{j\omega C_1}\frac{\partial\psi_1(x_1)}{dx_1} \quad \ldots \quad (10)$$

$$Z_1^2 = \frac{R_1 + j\omega L_1}{j\omega C_1} \quad \ldots \quad (11)$$

It should be noted that the constants P_1 and Z_1 depend only on the aerial, while the functions of x_1 depend on the constants of the aerial and the field distribution. The quantities $\mathbf{A}_1$ and $\mathbf{B}_1$ are constant vectors, in the same way that $\mathbf{i}_1$ and $\mathbf{v}_1$ are vectors.

This may seem rather heavy going for the not very mathematical reader, but the worst is over. The calculus is finished with at this point, and nothing more is required than a nodding acquaintance with $(a+jb)$ quantities, hyperbolic functions and elementary algebra.*

* See "Alternating Currents and Transients." Colebrook. (McGraw-Hill.)

The equations for the horizontal part will obviously be of the same form as for the vertical part, so we have finally

$$\mathbf{i}_1 = \mathbf{A}_1 \sinh P_1 x_1 + \mathbf{B}_1 \cosh P_1 x_1 + \psi_1(x) \quad (12)$$

$$\mathbf{v}_1 = -Z_1(\mathbf{B}_1 \sinh P_1 x_1 + \mathbf{A}_1 \cosh P_1 x_1) - \phi_1(x_1) \quad (13)$$

$$\mathbf{i}_2 = \mathbf{A}_2 \sinh P_2 x_2 + \mathbf{B}_2 \cosh P_2 x_2 + \psi_2(x_2) \quad (14)$$

$$\mathbf{v}_2 = -Z_2(\mathbf{B}_2 \sinh P_2 x_2 + \mathbf{A}_2 \cosh P_2 x_2) - \phi_2(x_2) \quad (15)$$

There appear to be four unknown constant vectors in these equations, but there are in fact only two, for they are not independent. At the point $x_1 = h_1$, $x_2 = 0$ we have $\mathbf{i}_1 = \mathbf{i}_2$ and $\mathbf{v}_1 = \mathbf{v}_2$, therefore

$$\mathbf{A}_1 \sinh P_1 h_1 + \mathbf{B}_1 \cosh P_1 h_1 + \psi_1(h_1) = \mathbf{B}_2 + \psi_2(0) \quad (16)$$

$$Z_1(\mathbf{A}_1 \cosh P_1 h_1 + \mathbf{B}_1 \sinh P_1 h_1 - \phi_1(h_1) = Z_2\mathbf{A}_2 - \phi_2(0) \quad (17)$$

so that $\mathbf{B}_2$ and $\mathbf{A}_2$ can be expressed in terms of $\mathbf{A}_1$ and $\mathbf{B}_1$. In addition there are two boundary conditions. At the foot of the aerial we have the tuning impedance $Z_r = R_r + jX_r$. Putting $\mathbf{i}_r$ and $\mathbf{v}_r$ for the current in the tuning impedance and the potential difference across it, then since $x_1 = 0$

$$\mathbf{i}_r = \mathbf{B}_1 + \psi_1(0) \quad \ldots \quad \ldots \quad (18)$$

$$\mathbf{v}_r = -Z_1\mathbf{A}_1 - \phi_1(0) \quad \ldots \quad \ldots \quad (19)$$

Therefore

$$\mathbf{A}_1 Z_1 + \phi_1(0) = \mathbf{B}_1 Z_r + \psi_1(0) Z_r \quad \ldots \quad (20)$$

Further, when $x_2 = h_2$, $\mathbf{i}_2 = 0$, that is

$$\mathbf{A}_2 \sinh P_2 h_2 + \mathbf{B}_2 \cosh P_2 h_2 + \psi_2(h_2) = 0 \quad (21)$$

From the four equations (16), (17), (20), and (21) it is simple algebra to determine $\mathbf{A}_1$ and $\mathbf{B}_1$, and then, from equation (18), to determine $\mathbf{i}_r$. This step need not be given in detail. The result is

$$\mathbf{i}_r = \frac{\phi_1(0) + \frac{N}{M} Z_1 \psi_1(0) - \frac{Q}{M} Z_1}{Z_r + \frac{N}{M} Z_1} \quad (22)$$

where tho following abbreviations have been used.

$$M = Z_2 \sinh P_1 h_1 \cosh P_2 h_2 + Z_r \cosh P_1 h_1 \sinh P_2 h_2 \quad (23)$$

$$N = Z_2 \cosh P_1 h_1 \cosh P_2 h_2 + Z_1 \sinh P_1 h_1 \sinh P_2 h_2 \quad (24)$$

$$Q = Z_2 \psi_2(h_2) + K_2 Z_2 \cosh P_2 h_2 + K_1 Z_1 \sinh P_2 h_2 \quad (25)$$

$$K_1 = \phi_1(h_1) - \phi_2(0) \quad \ldots \quad \ldots \quad (26)$$

$$K_2 = \psi_1(h_1) - \psi_2(0) \quad \ldots \quad \ldots \quad (27)$$

These expressions may seem somewhat complicated, but the actual form of the result is very simple, for

$$\mathbf{i}_r = \frac{\mathbf{e}_e}{Z_e + Z_r} \quad \ldots \quad \ldots \quad (28)$$

where

$$Z_e = \frac{N}{M} Z_1 \quad \ldots \quad \ldots \quad (29)$$

and

$$\mathbf{e}_e = \phi_1(0) + Z_e\{\psi_1(0) - Q/N\} \quad \ldots \quad (30)$$

Conclusions from the Form of the General Solution.

A number of useful conclusions can be drawn from the above without any further analysis.

(*a*) The effective impedance Z_e depends only on the constants of the aerial, being independent of Z_r and of the form of the field.

(*b*) The effective E.M.F. depends on the constants of the aerial and on the distribution of the fields.

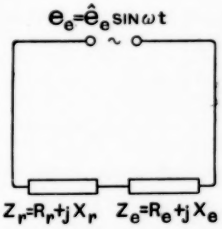


Fig. 2.

The complete circuit, consisting of the aerial with its distributed capacity, inductance and resistance and its distributed (not necessarily uniform) E.M.F.s and the tuning impedance, however constituted, can therefore be represented in the simple manner shown in Fig. 2, *i.e.*, as a simple series circuit. Of course, both the effective impedance of the aerial and the effective E.M.F. will vary with frequency in some more or less complicated manner to be considered later, but the simple equivalent quantities in Fig. 2 will accurately represent the state of affairs at any given frequency.

The effective height is clearly involved in the expression for the effective E.M.F. It should be noted that the effective height will depend on frequency, but will be independent of the tuning circuit.

Questions 1, 2 and 4 of the introduction have now been answered, at least in part, and the complete answers to the whole of the questions are contained in the above analysis.

Before considering the matter more in detail, however, one other point remains to be settled in relation to the general case. How does this reception of a distributed E.M.F. compare, from the point of view of the tuning or receiving circuit, with the behaviour of the aerial with respect to an E.M.F. concentrated at, say, the point $x_1 = 0$?

Considered as a special case of the general analysis, the solution can be obtained immediately by means of the appropriate substitutions, and the current through the tuning circuit impedance is given by

$$\mathbf{i}_r = \frac{\mathbf{e}}{Z_r + Z_e} \quad \ldots \quad \ldots \quad (31)$$

where Z_r and Z_e are as in the general case, and $\mathbf{e}$ is the vector representing the E.M.F. It may be thought that the substitution of a point E.M.F. for the continuous functions considered in the general case may not necessarily be legitimate. The writer has satisfied himself on this matter by retracing the steps of the analysis from the commencement. The conclusion is obviously consistent with the fact that the magnitude and distribution of the E.M.F. does not appear at all in the expression for the effective impedance.

This establishes a very useful fact, for it shows that, subject to the validity of the initial assumptions with regard to the general character of a receiving aerial, the effective aerial impedance and its variation with frequency can be studied experimentally by means of a local oscillator, with every assurance that the quantities so determined will be applicable to the behaviour of the aerial under any conditions of reception. The practical advantages of this are too obvious to need further comment.

Before considering the special cases and numerical quantities, further reference must be made to a point alluded to at the commencement of the analysis, *i.e.*, the possible non-uniformity of the capacity per unit length in the immediate neighbourhood of the earth connection.

The point has been considered in some

detail by Prof. Howe in his paper on the capacity of aerials.* He shows that the proximity effect is very local. It is nevertheless a matter for further investigation, and will be considered more fully at some future date. In any case, it is very unlikely that it will influence in any way the essential character of the results deduced, for even if the variation were so abrupt as to amount to a discontinuity, there is no reason to suppose that it would have any effect different in character from the discontinuity already considered at the junction between the vertical and horizontal parts of the aerial. Possibly the proximity effect could be exhibited as a small capacity permanently associated with the tuning circuit.

Further, some recent experiments carried out by Dr. Smith-Rose in conjunction with the present writer† have shown that a considerable part of the aerial resistance is associated with the earth connection, either as actual resistance or in the form of eddy current losses. This part of the total aerial resistance can be considered as permanently associated with the tuning circuit. Its effect will clearly be different from that of the distributed resistance to which the above analysis refers. This will not affect the analysis at all, beyond setting a limit beyond which the resistance of the tuning impedance cannot be reduced.

We are now in a position to consider certain important practical special cases, with a view to bringing the general solution into a more physically comprehensible form and, if possible, substituting actual quantities for some of the symbols.

1. If the component parts of the aerial are straight lines and the aerial is situated in a uniform (not necessarily vertical) field, then $\mathbf{e}_1$ and $\mathbf{e}_2$ are constants with respect to the current co-ordinates, so that

$$\psi_1(0) = \psi_1(h_1) = \mathbf{e}_1/P_1Z_1 \quad \ldots \quad (32)$$

$$\psi_2(0) = \psi_2(h_2) = \mathbf{e}_2/P_2Z_2 \quad \ldots \quad (33)$$

and

$$\phi_1(x_1) = \phi_2(x_2) = 0 \quad \ldots \quad \ldots \quad (34)$$

The substitution of these values in the general solution will give

$$\mathbf{e}_e = \left[\frac{\mathbf{e}_1}{P_1Z_1} - \frac{1}{N}\left\{\frac{\mathbf{e}_2}{P_2} + Z\cosh Ph\left(\frac{\mathbf{e}_1}{P_1Z_1} - \frac{\mathbf{e}_2}{P_2Z_2}\right)\right\}\right]Z_e \quad (35)$$

and

$$Z_e = \frac{Z_1 + Z_2 \coth P_1h_1 \coth P_2h_2}{Z_1 \coth P_1h_1 + Z_2 \coth P_2h_2} \quad (36)$$

2. If, in addition, the aerial is uniform throughout in its electrical constants, *i.e.*, if both parts are composed of the same conductors similarly disposed or a single conductor, then we can put $Z_1=Z_2=Z$ and $P_1=P_2=P$, in which case the above expressions can be further reduced to

$$\mathbf{e}_e = \frac{1}{P}\frac{\mathbf{e}_1(\cosh Ph - \cosh Ph_2) + \mathbf{e}_2(\cosh Ph_2 - 1)}{\sinh Ph} \quad (37)$$

and

$$Z_e = Z \coth Ph \quad \ldots \quad (38)$$

where

$$h = h_1 + h_2 \quad \ldots \quad (39)$$

3. If the vertical and horizontal parts of the aerial are dissimilar, *i.e.*, have different constants, but the vertical part is parallel to a uniform field and the horizontal part perpendicular to it, then $\mathbf{e}_1=\mathbf{e}$ and is constant with respect to x_1, while $\mathbf{e}_2=0$. Then

$$\mathbf{e}_e = \frac{\mathbf{e}}{P_1}\left\{\frac{Z_2\cosh P_2h_2(\cosh P_1h_1 - 1) - Z_1\sinh P_2h_2\sinh P_1h_1}{Z_2\sinh P_1h_1\cosh P_2h_2 - Z_1\cosh P_1h_1\sinh P_2h_2}\right\} \quad (40)$$

and Z_e will be the same as in case 1.

4. If in addition to the above the vertical and horizontal parts of the aerial have the same constants, then

$$\mathbf{e}_e = \frac{\cosh Ph - \cosh Ph_2}{\sinh Ph}\frac{\mathbf{e}}{P} \quad \ldots \quad (41)$$

and

$$Z_e = Z \coth Ph \quad \ldots \quad (42)$$

where

$$h = h_1 + h_2$$

5. Finally, for the simplest case of all (scarcely ever seen outside a text-book), *i.e.*, a plain straight aerial parallel to a uniform vertical field, $\mathbf{e}_1=\mathbf{e}$ (const.), $h_2=0$, giving

$$\mathbf{e}_e = \frac{\cosh Ph - 1}{\sinh Ph}\frac{\mathbf{e}}{P} \quad \ldots \quad (43)$$

$$= \frac{\mathbf{e}}{P}\tanh\frac{Ph}{2} \quad \ldots \quad (44)$$

and

$$Z_e = Z \coth Ph \quad \ldots \quad (45)$$

*"On the Capacity of Radio-Telegraphic Antennæ." Prof. G. W. O. Howe. *Wireless World*, Vol. II., pp. 546, 612, 680.

†"Some Experiments with Aerial and Earth Circuits." *E.W. & W.E.*, Vol. II., No. 16, p. 207, January, 1925.

Reduction of the Solutions to Scalar Form.

The foregoing expressions for the various special cases are not yet in a form suitable for numerical calculation, or for exhibiting the effect of the various constants of the aerial structure. For this purpose it will be necessary to reduce to the form $(a+jb)$ or $re^{j\theta}$ the various complex functions of the aerial constants involved in the expressions for Z_e and $\mathbf{e}_e$.

This part of the work will be confined to the cases in which both parts of the aerial have the same electrical constants. The methods involved will be equally applicable to the general case, but the latter is so much more cumbersome in form that the main conclusions are likely to be obscured by the bulk of the expressions.

The following resolutions of Ph and Z into their component parts presents no difficulties.

$$P^2 = (R + j\omega L)\, j\omega C \quad \ldots \quad (46)$$

Therefore

$$(Ph)^2 = (Rh + j\omega Lh)\, j\omega Ch \quad \ldots \quad (47)$$

$$= (R_0 + j\omega L_0)\, j\omega C_0 \quad \ldots \quad (48)$$

where R_0, L_0, and C_0 are the total resistance, inductance and capacity of the aerial. If Ph be expressed in the form $A+jB$ (the A's and B's of the former part are now finished with, so no ambiguity will arise), then since

$$(A + jB)^2 = (R_0 + j\omega L_0)\, j\omega C_0 \quad \ldots \quad (49)$$

$$A^2 - B^2 = -\omega^2 L_0 C_0 \quad \ldots \quad (50)$$

$$AB = \omega C_0 R_0/2 \quad \ldots \quad (51)$$

whence

$$A^2 + B^2 = \omega C_0 \sqrt{R_0^2 + \omega^2 L_0} \quad (52)$$

On the assumption that $R_0^4/\omega^4 L_0^4$ is negligibly small compared with 1, the above equations will give for A and B

$$A = \frac{R_0}{2}\sqrt{\frac{C_0}{L_0}} \quad \ldots \quad \ldots \quad \ldots \quad (53)$$

$$B = \omega\sqrt{L_0 C_0}\left(1 + \frac{R}{8\omega^2 L_0^2}\right) \quad \ldots \quad (54)$$

Also

$$Z = P/j\omega C \quad \ldots \quad \ldots \quad (55)$$

$$= Ph/(j\omega C_0) \quad \ldots \quad \ldots \quad (56)$$

$$= \frac{B}{\omega C_0} - j\frac{A}{\omega C_0} \quad \ldots \quad (57)$$

and substituting the values for A and B

$$Z = \sqrt{\frac{L_0}{C_0}}\left(1 + \frac{R_0^2}{B\omega^2 L_0^2}\right) - j\frac{R}{2}\sqrt{\frac{1}{\omega^2 L_0 C_0}} \quad (58)$$

Thus both Ph and Z are known in terms of the constants of the aerial.

It will now be well to consider a typical small receiving aerial in order to see what values these quantities may have in practice. We will assume the following dimensions :—

h_1 = 30 ft. = 915 cms.

h = 70 ft. = 2134 cms.

$h = h_1 + h_2 = 100$ ft. = 3049 cms.

radius of wire = .108 cms. (abt. 3/19)

The inductance and capacity per unit length can be calculated from the formulæ

$$C = (4.606\, log_{10}(h/r) - .614)^{-1} \times 1.111 \quad \mu\mu\text{F/cm.}$$

$$L = (4.606\, log_{10}(h/r) - .614)\, 10^{-9} \text{ H/cm.}$$

These will give

$L = 19.887 \times 10^{-9}$ H/cm. $L_0 = 60.64\mu$H.

$C = .056\ \mu\mu$F/cm. $C_0 = 170.7\mu\mu$F.

The resistance R_0 cannot very well be calculated, and very little data is available for an estimate. To be on the safe side in the matter of neglecting resistance terms in subsequent approximations, we will take $R_0 = 50$, which is likely to be a liberal estimate (it must be remembered that eddy current and earth connection losses are not included in this figure, these being considered as permanently associated with the tuning impedance).

From equations (61-63),

$$A = \frac{R}{2}\sqrt{\frac{C_0}{L_0}} \text{ (very approx.)}$$

$$= .042$$

In general A will be a small quantity, less than .1. The value of B will of course depend on the frequency at which the aerial is actually being used. Thus, for a wavelength of 365 metres,

$$B = \omega^2 L_0 C_0 \text{ (approx.)}$$

$$= .524.$$

It will be shown later that $B = 2\pi h/\lambda$ to a very close approximation, λ being the wavelength of operation.

We will now consider the resolution of the impedance expression into its resistance and reactance components.

$$Coth\ Ph = coth\ (A + jB) \quad \ldots \quad \ldots \quad (59)$$

$$= \frac{sinh\ 2A - j\ sin\ 2B}{cosh\ 2A - \ cos\ 2B} \quad \ldots \quad (60)$$

and since

$$Z_e = Z \coth Ph \quad \ldots \quad \ldots \quad (61)$$

$$= \frac{1}{\omega C_0}(B - jA) \coth Ph \quad \ldots \quad (62)$$

$$= R_e + jX_e \quad \ldots \quad \ldots \quad (63)$$

we have

$$R_e = \frac{1}{\omega C_0}\left(\frac{B \sinh 2A - A \sin 2B}{\cosh 2A - \cos 2B}\right) \quad (64)$$

$$X_e = -\frac{1}{\omega C_0}\left(\frac{A \sinh 2A + B \sin 2B}{\cosh 2A - \cos 2B}\right) \quad (65)$$

For all except very inefficient aerials it will be permissible to put

$$\sinh 2A = 2A \quad \ldots \quad \ldots \quad (66)$$

$$\cosh 2A = 1 \quad \ldots \quad \ldots \quad (67)$$

so that the above expressions become

$$R_e = \frac{A}{\omega C_0}\left(\frac{2B - \sin 2B}{1 - \cos 2B}\right) \quad \ldots \quad (68)$$

$$X_e = -\frac{1}{\omega C_0}\left(\frac{2A^2 + B \sin 2B}{1 - \cos 2B}\right) \quad \ldots \quad (69)$$

The above expressions are in quite a simple form for calculation, but can be simplified still further if, as is usually the case, the aerial is being used at a wavelength which is long compared with its fundamental. The following table will be a guide in this matter.

$\lambda/\lambda_0 > 3.5$ series up to 5th power correct to .1 per cent.

series up to 3rd power correct to .5 per cent.

$\lambda/\lambda_0 > 2.4$ series up to 5th power correct to about 3 per cent.

In the above the series referred to is the series form for the sin and cosine of the angle $2B$, and λ_0 is the natural wavelength of the aerial.

Using the series form for *sin* and *cos* $2B$ up to the fifth power of the angle the expression for R_e reduces to

$$R_e = \frac{2AB}{3\omega C_0}(1 + 2B^2/15) \quad \ldots \quad (70)$$

and since

$$2AB = \omega C_0 R_0 \quad \ldots \quad (71)$$

and

$$B^2 = \omega^2 L_0 C_0 \text{ very approx.} \quad \ldots \quad (72)$$

$$R_e = \frac{R_0}{3}(1 + \tfrac{2}{15}\omega^2 L_0 C_0) \quad \ldots \quad (73)$$

In the same way the expression for the reactance reduces to

$$X_e = -\frac{1}{\omega C_0}\left(1 - \frac{\omega^2 L_0 C_0}{3} - \frac{\omega^4 L_0^2 C_0^2}{45}\right) \quad (74)$$

to the same degree of approximation.

Thus, for wavelengths two or three times the natural wavelength of the aerial, the latter behaves very nearly as shown in Fig. 3, *i.e.*, as a resistance in series with a capacity since the bracket terms are very nearly unity. *A standard aerial can therefore be represented very closely in this way over the broadcasting range of wavelengths.*

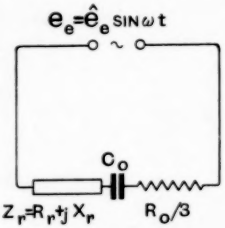


Fig. 3.

Taking as an example the typical case quoted above, and assuming a wavelength of 365 metres, then

$$R_e = \frac{50}{3}(1.037)$$

$$= 17.28 \text{ ohms.}$$

$$X_e = -1{,}135\ (1 - .092 - .0014)$$

$$= -1{,}135 \times .907$$

$$= -1030 \text{ ohms.}$$

Actually an aerial of this description would probably appear to have a resistance of 30 or 40 ohms, but the use of an earth screen would reduce this figure to about 15 ohms or so, showing that a considerable part of the apparent resistance is attributable to eddy current and other losses at the earth connection.

Resonance. Natural Wavelength.

Expressing the effective impedance in the form

$$Z_e = \mathbf{z}_e\, \varepsilon^{j\theta} \quad \ldots \quad \ldots \quad (75)$$

the resonant frequency can be defined by

$$d\,\mathbf{z}_e/d\omega = 0 \quad \ldots \quad \ldots \quad (76)$$

or more conveniently in practice by

$$d\mathbf{z}_e^2/d\omega = 0 \quad \ldots \quad \ldots \quad (77)$$

However, the differentiation with respect to ω will be laborious and cumbersome, and a better way of arriving at the resonance relation between ω and h will be to consider the frequency as a constant and find the corresponding critical value of h.

From equations (68) and (70),

$$\mathbf{z}_e^2 = \frac{A^2 + B^2}{\omega^2 C_0^2} \frac{\cosh 2A + \cos 2B}{\cosh 2A - \cos 2B} \quad (78)$$

(This transformation is not given in detail as it is a standard one.) Equating to zero the differential coefficient with respect to h gives

$$(A' \sin 2A - B' \sin 2B)(\cosh 2A - \cos 2B) - (A' \sinh 2A + B' \sin 2B)(\cosh 2A + \cos 2B) = 0 \quad (79)$$

where A' has been written for dA/dh, etc. and since A is proportional to h,

$$A' = A/h \quad \ldots \quad \ldots \quad (80)$$

Equation (79) reduces to

$$A \tanh 2A + B \tan 2B = 0 \quad \ldots \quad (81)$$

which is therefore the equation defining the resonance relation between ω and h.

If A is negligibly small this becomes

$$B \tan 2B = 0 \quad \ldots \quad (82)$$

which gives the well-known result for the fundamental

$$\omega_0^2 L_0 C_0 = \pi^2/4 \quad \ldots \quad (83)$$

For given values of A and B the solution of the resonance equation could be found by means of tables, but since A will always be small, the following process will give a close idea of the effect of the resistance on the natural wavelength of the aerial. If we put

$$2B = (\pi + \alpha) \quad \ldots \quad (84)$$

where α is some unknown small angle, it is easy to derive from 81, substituting $2A$ for $A \tanh 2A$, the equation

$$\alpha^2 + \alpha\pi + A^2 = 0 \quad \ldots \quad (85)$$

so that

$$\alpha = -A^2/\pi \text{ (approx.)} \quad \ldots \quad (86)$$

and since

$$B^2 = A^2 + \omega_0^2 L_0 C_0 \text{ (see eq. (61), } \omega = \omega_0) \quad (87)$$

therefore

$$\omega_0 = \frac{\pi}{2\sqrt{L_0 C_0}} \left(1 - \frac{3A^2}{\pi^2}\right) \quad \ldots \quad (88)$$

$$= \frac{\pi}{2\sqrt{L_0 C_0}} \left(1 - \tfrac{3}{4} \frac{C_0 R_0^2}{L_0}\right) \text{ (approx.)} \quad (89)$$

Now $L_0 C_0 = h^2/u^2$ (very approximately) (90)

(see formulæ for L and C)

where

$$u = 3 \times 10^{10} \text{ cms/sec.} \quad \ldots \quad (91)$$

and

$$\omega_0 = 2\pi u/\lambda_0 \quad \ldots \quad (92)$$

where λ_0 is the natural wavelength of the aerial.

Substituting these values in the equation for ω_0 gives

$$\lambda_0 = 4h\left(1 + \frac{3}{4 \times 9} \frac{h^2 R_0^2}{L_0^2} 10^{-20}\right) \quad \ldots \quad (93)$$

$$= 4h\left(1 + \frac{h^2 R_0^2}{12 L_0^2} 10^{-20}\right) \quad \ldots \quad (94)$$

Thus the effect of the resistance of the aerial is seen to be a very small increase in the natural wavelength of the aerial. In general, however, the effect will be negligibly small. For instance, taking the figures of the typical case quoted above, we have

$$\lambda_0 = 4h(1.005)$$

a difference of only five parts in a thousand.

For all practical purposes, therefore, we may take the natural wavelength as four times the total length (*i.e.*, $h_1 + h_2$) of the aerial. Therefore

$$B^2 = \omega^2 L_0 C_0 \quad \ldots \quad (95)$$

$$= \frac{\pi^2 \lambda_0^2}{4\lambda^2} \quad \ldots \quad \ldots \quad (96)$$

$$= \frac{4h^2\pi^2}{\lambda^2} \quad \ldots \quad \ldots \quad (97)$$

and equations (73) and (74) for the effective resistance and reactance can be put in the forms

$$R_e = \frac{R_0}{3}\left(1 + \frac{8\pi^2 h^2}{15\lambda^2}\right) \quad \ldots \quad (98)$$

$$X_e = -\frac{1}{\omega C_0}\left(1 - \frac{4\pi^2 h^2}{3\lambda^2} - \frac{16\pi^2 h^2}{45\lambda^2}\right) \quad (99)$$

Effective Height.

Of the questions put in the introduction the first three are now answered completely. The remaining two will now be considered.

For simplicity we will consider first the somewhat academic case of a plain aerial, vertical, and parallel to a uniform field. (See equations (44) and (45).) The effective E.M.F. has been shown to be

$$\mathbf{e}_e = \frac{h}{Ph} \tanh \frac{Ph}{2} \mathbf{e} \quad \ldots \quad (100)$$

where $\mathbf{e}=\hat{E}\ sin\ \omega t$ is the uniform field intensity. Putting this in the form

$$\mathbf{e}_e = H\epsilon^{j\varphi}\mathbf{e} \quad \ldots \quad \ldots (101)$$

then H is the effective height of the aerial, since HE is the amplitude of the total E.M.F. acting in series with the effective aerial impedance and the tuning impedance. From equation (100)

$$H^2 = \frac{h^2}{A^2+B^2}\,\frac{cosh\,A - cos\,B}{cosh\,A + cos\,B} \quad \ldots (102)$$

(This transformation is not given in detail as it can easily be verified.)

In nearly all practical cases, it will be permissible to put $cosh\ A=1$ to an accuracy better than 1 per cent, so that

$$H^2 = \frac{h^2}{A^2+B^2}\,\frac{1 - cos\,B}{1 + cos\,B} \quad \ldots (103)$$

$$= \frac{h^2}{A^2+B^2}\,tan^2\,B/2 \quad \ldots (104)$$

therefore

$$H = \frac{h}{\sqrt{A^2+B^2}}\,tan\,B/2$$

This is in a form quite convenient for calculation without further approximation. For instance, assuming that the typical aerial already considered is wholly vertical,

$$H = \frac{tan\ (.5244\ \text{radians}/2)}{\sqrt{.042^2 + .5244^2}}\,h.$$

$$= .512\,h$$

To show better the character of the expression in the case where A^2 is negligible compared with B^2, and where λ is two or three times the natural wavelength of the aerial, the tangent can be expressed in series form up to, say, the fifth power, giving

$$H = \frac{h}{B}\left(\frac{B}{2}+\frac{B^3}{24}+\frac{B^5}{240}\right) \quad \ldots \quad \ldots (105)$$

$$= \frac{h}{2}\left(1 + \frac{B^2}{12}+\frac{B^4}{120}\right) \quad \ldots \quad \ldots (106)$$

$$= \frac{h}{2}\left(1 + \frac{\pi^2}{48}\,\frac{\lambda_0^2}{\lambda^2}+\frac{\pi^4}{1920}\,\frac{\lambda_0^2}{\lambda^2}\right) \quad (107)$$

$$= \frac{h}{2}\left(1 + \frac{\pi^2}{3}\,\frac{h^2}{\lambda^2}+\frac{2}{15}\,\frac{\pi^4 h^4}{\lambda^4}\right)^* \quad (108)$$

Thus in general H is very nearly equal to $h/2$.

* In Mr. Moullin's paper this is given to the first two terms as $\frac{h}{2}\left(1 + \frac{\pi^2 h^2}{4\lambda^2}\right)$ but this is an obvious slip.

A much more important case in practice is that corresponding to the typical aerial considered as a numerical example, *i.e.*, the case in which the aerial is partly vertical and partly horizontal, the vertical part being parallel to a uniform field. (See equation (52).) The effective E.M.F. is given by

$$\mathbf{e}_e = \frac{cosh\,(A+jB) - cosh(A_2+jB_2)}{sinh\,(A+jB)}\,\frac{h}{Ph}\,\mathbf{e} \quad (109)$$

where

$$Ph_2 = A_2 + jB_2 \quad \ldots \quad (110)$$

The derivation from this expression of the value of H is somewhat lengthy, but follows exactly the same lines as in the simpler case, and need not be given in detail. The result is

$$H^2 = (2h^2/A^2+B^2)\,\frac{\{cosh(A+A_2) - cos\,B + B_2)\}\{cosh(A-A_2) - cos(B-B_2)\}}{(cosh\,2A - cos\,2B)} \quad (111)$$

If A is small and A^2 negligible compared with B^2, this reduces to

$$H^2 = \frac{2h^2}{B^2}\,\frac{(1 - cos\,\overbrace{B+B_2})\,(1 - cos\overbrace{B-B_2})}{(1 - cos\,2B)} \quad (112)$$

whence

$$H = \frac{h}{B}\,\frac{cos\,B - cos\,B_2}{sin\,B} \quad \ldots \quad (113)$$

For the numerical case already considered

$$B = 2.544\ \text{radians} = 30^\circ\,3'$$

$$B_2 = .7 \times .2544\ \text{radians}$$

$$= .367\ \text{radians}$$

$$= 21^\circ$$

and the substitution of these values in the formula gives

$$H = .266\,h$$

and since

$$h_1 = .3\,h$$

it will be seen that the effect of the horizontal part is to make the effective height very nearly equal to the actual vertical height. This shows the advantage of having a fairly long horizontal part to the aerial in cases where the vertica height is limited. It also shows that in the above typical case very little is to be gained by doubling or trebling the top wires, certainly not much more than about 10 per cent. This conclusion is quite in agreement with measurements made on small receiving aerials by the writer in conjunction with Dr. Smith-Rose, described in the article referred to above.

Moreover, the form of the expression for the vertical height shows clearly that the best distribution of a given total length of aerial is to have the whole length vertical. That is to say, there is no best distribution as between vertical and horizontal parts for a given total length. This is illustrated by the curve of Fig. 4, which shows effective height plotted against h_2 for the numerical case considered. This provides the answer to Question 5.

the case described above the following formulæ are applicable if the wavelength is two or three times that of the aerial.

$$R_e = \frac{R_0}{3}\left(1 + \frac{8}{15}\frac{\pi^2 h^2}{\lambda^2}\right)$$

$$X_e = -\frac{1}{\omega C_0}\left(1 - \frac{4}{3}\frac{\pi^2 h^2}{\lambda^2} - \frac{16}{45}\frac{\pi^4 h^4}{\lambda^4}\right)$$

The accurate formulæ for the general case are given in equations (64) and (65).

2. *Is the effective impedance dependent on (a) the nature of the tuning or receiving circuit; (b) the distribution of the electric field to the signal?*

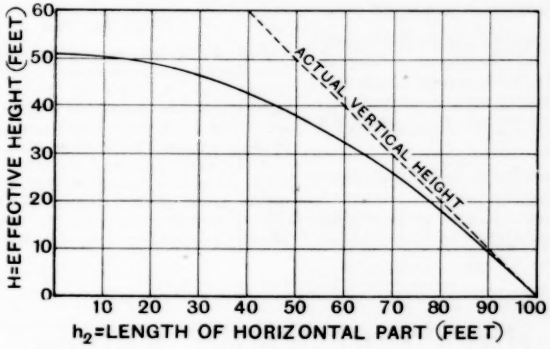


Fig. 4.

SUMMARY.

It will be well to conclude the paper with a restatement of the initial questions giving to each the answer that has been found in the course of the analysis. The answers will be given in as widely applicable a form as possible, but for convenience some of the more important specific formulæ will be quoted. Unless otherwise stated these refer to an aerial of height h_1 (vertical) with a horizontal top of length h_2 situated in a uniform vertical field. The vertical and horizontal parts of the aerial are considered to have the same uniform electrical constants L, C, and R per unit length, the total resistance inductance and capacity, *i.e.*, $R(h_1+h_2)=Rh$, etc., being represented by R_0, L_0, and C_0. The natural wavelength of the aerial will be called λ_0.

1. *What is the nature of the effective impedance of a receiving aerial from the point of view of associated receiving apparatus?*

From the point of view of an inserted receiving or tuning circuit the aerial can be regarded as an ordinary impedance of the type (R_e+jX_e) in series with a certain effective E.M.F. due to the signal. The resistance component of the aerial impedance will not vary greatly with frequency if the wavelength of the signal is two or three times the natural wavelength of the aerial. Under the same conditions the effective reactance can be represented very closely as that of the total aerial capacity. For

The effective impedance depends *only* on the electrical constants and the configuration of the aerial.

3. *What part is played by the distributed resistance of the aerial?*

The distributed resistance of the aerial enters into the resistance component of the effective impedance and also, though not considerably, into the effective reactance. It is one of the factors which limits the current at the base of the aerial when the latter is tuned to resonance with the signal E.M.F. It is, however, only one of the factors which limits the resonance current, the others (eddy current and other losses in the earth connection and the resistance of the tuning impedance) being in general of at least equal and probably greater effect. (For formulæ see Question 1.)

4. *What is the effective height of an aerial; and does it depend on (a) the tuning circuit conditions, (b) the field distribution?*

The aerial circuit can be shown to be equivalent to a certain effective E.M.F. in series with the effective aerial impedance and the tuning circuit impedance. The term, effective height, can only be applied if the field can be considered uniform. The effective height can then be defined as the magnitude of the total E.M.F. divided by the magnitude of the field intensity. Thus, if the effective E.M.F. is given by

$$\mathbf{e}_e = H_e e^{j\varphi}\mathbf{e}$$

where **e** is the field intensity, then H is the effective height for reception.

For a plain vertical aerial in a uniform vertical field—

$$H^2 = \frac{h}{A^2 + B^2} \cdot \frac{\cosh A - \cos B}{\cosh A + \cos B}$$

where

$$A = \frac{R}{2}\sqrt{\frac{C_0}{L_0}}$$

$$B = \omega\sqrt{L_0 C_0}\left(1 + \frac{R_0^2}{8\omega^2 L_0^2}\right)$$

assuming that $R_0^4/\omega^4 L_0^4$ is negligibly small compared with (1). In practice H will be very nearly to $h/2$. Other formulæ are given in equations (111) and (113).

The effective height is not effected by the tuning of the aerial, but depends on the frequency and on the field distribution. It is thus not a specific constant for any given aerial structure, but can be made so for purposes of definition by assuming a uniform vertical field.

5. *Is there any optimum distribution for a given length of aerial?*

No perfectly general answer can be given to this question since the effective height and therefore the effectiveness of the aerial depend on the configuration of the field in which it is situated. In the case of a uniform vertical field, however, the best possible arrangement for a given total length is wholly vertical.

In conclusion, the writer would like to point out that though the above conclusions are entirely consistent with his own experience with small receiving aerials, there is clearly need for experimental confirmation or otherwise of much of the detail, and also of the extent to which the assumptions with regard to the effective uniformity of the aerial constants are really valid in practice. It is partly in the hope of stimulating such experimental work that the above paper has been published.

NOTE BY THE AUTHOR.

This article was written nearly two years ago but I have not had occasion to modify any of the conclusions reached in the light of further consideration or experience. Moreover, a certain amount of experimental work on this subject has recently been carried out at the National Physical Laboratory, by Mr. Wilmotte, and the results obtained are substantially in agreement with the main deductions of the theoretical investigation given above. It is hoped that some account of Mr. Wilmotte's work on the subject will shortly be available for publication.—F. M. COLEBROOK.

X-Rays and Radio Valves.

By J. Taylor, D.Sc. (Utrecht), M.Sc., Ph.D., A.Inst.P.

WHEN anybody mentions X-rays we usually think at once of the radiation coming from great bulbs driven by some fifty or more kilovolts, high power installations such as are used for radiographic and medical purposes.

Such radiations are what is technically termed "hard," and are not readily absorbed in substances. Their wavelength is of the order of 10^{-8} cms.—that is, one hundredth of a millionth of a cm. Ordinary visible light on the other hand has a wavelength of the order of 5×10^{-5} cms.—fifty millionths of a cm. We thus see immediately what a large difference of magnitude exists between the two types of waves.

In a general way it is found that the greater the voltage driving an X-ray bulb the "harder" the radiation emitted from it, or in other words, the smaller the wavelength of the radiation. Indeed the frequency of the hardest radiation from a tube is given by the quantum relation $\eta = V/c$, where V is the voltage across the bulb and c is a constant. Alternatively we may express this relation in terms of wavelengths and obtain the relation, $V \times \lambda = 12{,}340$, where λ is the wavelength of the hardest radiation, expressed in ångstrom units (the usual unit for ordinary radiation wavelengths).

It is possible by utilising X-ray bulbs of the Coolidge type in which the cathode consists of a glowing filament of tungsten placed very near to an anticathode or target—suitably of tungsten—mounted in a highly exhausted vessel, to obtain X-radiation with very much smaller voltages across the tube. In this way X-rays corresponding to three or four hundred volts may be examined—that is of the order of 50×10^{-8} cms. (50 ångstroms). Such radiations are characterised by great absorbability, thin films of celluloid of as little as ten millionths of a cm. in thickness, absorbing 90 per cent. of the radiation. The rays are consequently completely absorbed in the walls of the bulb and their

properties cannot be examined outside in the surrounding space. Several methods, however, can be utilised to measure some of their properties, within the tube itself.

When the radiation passes through gases strong ionisation is produced, the gas molecules and atoms being split up into

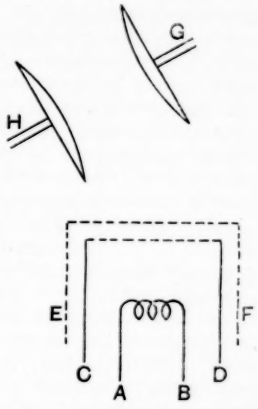


Fig. 1. *Type of tube shown diagrammatically.*

positive and negative parts—that is positive ions, and negative ions and electrons—the gases become to a certain extent electrically conducting and if suitable means are taken, a current passing through the gas may be measured. Also, if the radiations fall against a negatively charged plate, electrons (particles of negative electricity) are given off or expelled from the surface of the metal due to the photo-electric action of the radiation upon it. Either of the above described actions may be utilised for the detection and measurement of the soft X-rays.

When such low voltage X-ray bulbs are used it is necessary to maintain a very high vacuum within them, and to freeze out by means of a liquid air trap all traces of mercury vapour (mercury is almost universally used in high vacuum pumps and so is always present in the apparatus unless special precautions are taken to freeze it out).

If traces of gas or mercury vapour—even at a pressure of less than a millionth of an atmosphere—remain in the tube, the type of radiation emitted becomes immediately softer—that is of greater wavelength—because it is then to a large extent produced by the impact of the electrons against the gas or vapour molecules in the tube, or absorbed as a thin film upon the anti-cathode surface, and the radiation accompanying such collisions corresponds to teens of volts instead of to a few hundred volts. At the same time, however, the output of radiations or total intensity is very much increased.

We may state then that it is a general property that when electrons strike against a metal target radiation is emitted, or when they impinge against gas molecules and have sufficient energy, they produce not only ionisation of the gas but give rise to radiations from the gas atoms and molecules.

In principle then every diode or triode valve is a generator of X-radiation of long wavelength. The quantity generated may, of course, be very small but it nevertheless must exist.

The writer has recently carried out experiments which show this property. Fig. 1 shows diagrammatically the type of tube employed. AB is a tungsten filament, CD a nickel cylinder provided with a nickel gauze window at its upper end and surrounding the filament, EF is a gauze grid which completely encloses the cylinder

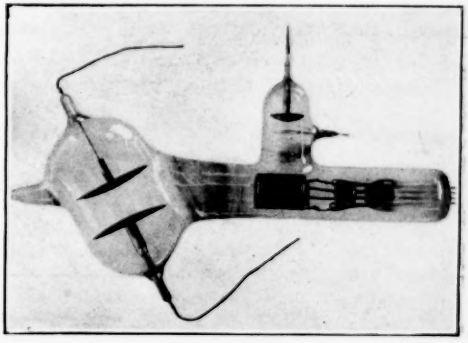


Fig. 2. *Photograph of the tube showing position of the electrodes.*

system. CD is utilised as plate and the electron current flows between AB and CD. EF is charged to such a potential as to prevent any escape of electrons or ions from the cylinder-filament system. The two metal (nickel) electrodes G and H are

situated as shown in the photograph, Fig. 2, and EF, so that G is in view of the opening or window in the cylinder CD. The apparatus was exhausted to as high a vacuum as possible and thoroughly "baked out" and "glowed out" for many hours. A liquid air trap was also used.

The X-radiation was detected and measured by means of the photo-electric effect produced at the electrode G. The photo-electric currents were measured by means of a sensitive galvanometer. It was found that no current flowed between G and the filament AB provided that G was at a fairly high positive potential with respect to the filament. This shows definitely, therefore, that no electrons were gathered from the filament by the electrode G. When G became of the order of some volts positive however a small current was indicated and this current increased as the potential of G was increased in the negative direction, until finally a saturation value was reached.

Similar results were obtained by measuring the current between the electrodes G and H. The maximum currents obtained were of the order of about 5×10^{-10} amps.

There is no reasonable doubt but what the currents were produced by the photo-electric action of radiation proceeding from the bombardment of the cylinder by the filament electrons, for further investigation showed that the current exhibited the properties of photo-electric emissions and was definitely not due to the photo-electric action of the light emitted by the filament.

When the liquid air was removed so that mercury vapour—the vapour of mercury at room temperature has a pressure of less than a millionth of an atmosphere—entered the tube an increase of some ten or fifteen fold in the photo-electric current occurred due to the soft radiation produced by the bombardment of the electrons against the mercury atoms and to a certain extent the creation of a positive space charge.

These experiments show definitely, as indeed previous experiments along similar lines have done, that there must be production of soft X-radiation within the diode and triode valves used in practice. This soft X-radiation is intensely ionising and can produce considerable photo-electric effects on metallic and other substances. In hard valves such radiation must exist though most probably in small amount. In soft valves, however, there is a possibility of considerable production of radiation which will in turn bring about photo-electric emissions from the cathode—filament—and ionisation within the gas. When traces of substances such as the alkali metals which exhibit very great photo-electric emissivity are included in the tubes such photo-electric effects may conceivably become important.

The Performance of Valves in Parallel.

By R. P. G. Denman, M.A., A.M.I.E.E.

THERMIONIC valves are commonly used in parallel as a convenient method of obtaining large power output without the necessity for designing special valves for every requirement. It is evident from the performance of large banks of valves that good efficiencies can be obtained, but in view of the fact that no two valves are ever likely to have precisely similar characteristics, it may be of interest to examine the general case of a number of valves operating in parallel on a common load. We shall then be in a position to judge the extent of the losses which are liable to occur in practice, and decide how far it may be necessary, or possible, to redress the balance by means of separate grid bias, etc., for individual valves.

1. Theoretical Case of n Batteries in Parallel.

We will begin by establishing one or two formulæ concerning the general type of network shown in Fig. 1. Applying Kirchhoff's First and Second Laws we have :—

$$\begin{aligned} I &= i_1 + i_2 + i_3 + \ldots + i_n \\ V_1 &= i_1 r_1 + IR_e \\ V_2 &= i_2 r_2 + IR_e \\ &\ldots\ldots \\ V_n &= i_n r_n + IR_e \end{aligned}$$

$$\therefore \quad i_1 = \frac{V_1}{r_1} - \frac{IR_e}{r_1} \qquad \ldots \quad (1)$$

and similar equations, which added together give :—

$$I = \left(\frac{V_1}{r_1} + \frac{V_2}{r_2} + \ldots + \frac{V_n}{r_n}\right) - IR_e\left(\frac{1}{r_1} + \frac{1}{r_2} + \ldots \frac{1}{r_n}\right) \qquad (2)$$

Designating this as

$$I = \Sigma^n \frac{V}{r} - IR_e\Sigma^n \frac{1}{r} \qquad \ldots \quad (3)$$

we obtain

$$I = \frac{\Sigma^n \frac{V}{r}}{1 + R_e \Sigma^n \frac{1}{r}} \qquad \ldots \quad (4)$$

Substituting this value for I in equation (1) we get :—

$$i_1 = \frac{V_1 - IR_e}{r_1} = \frac{V_1}{r_1} - \frac{R_e\Sigma^n(V/r)}{r_1(1 + R_e\Sigma^n(1/r))}$$

$$i_2 = \frac{V_2 - IR_e}{r_2} = \frac{V_2}{r_2} - \frac{R_e\Sigma^n(V/r)}{r_2(1 + R_e\Sigma^n(1/r))}$$

In the special case $n=2$ we find that

$$I = \frac{V_1 r_2 + V_2 r_1}{r_1 R_e + r_2 R_e + r_1 r_2} \qquad \ldots \quad (5)$$

$$i_1 = \frac{V_1 R_e + V_1 r_2 - V_2 R_e}{r_1 R_e + r_2 R_e + r_1 r_2} \qquad \ldots \quad (6)$$

$$i_2 = \frac{V_2 R_e + V_2 r_1 - V_1 R_e}{r_1 R_e + r_2 R_e + r_1 r_2} \qquad \ldots \quad (7)$$

It is to be noted that i_2 becomes zero when

$$R_e = \frac{V_2}{V_1 - V_2} r_1$$

and negative when R_e exceeds this value. The condition that any current is positive is that

$$V_n > IR_e$$

Thus if R_e increases until the current in one of the branches becomes zero, the first to suffer will be that having the smallest E.M.F.

Now let the load R_e be short-circuited. Then the total current I (equation (4)) is

$$I = \Sigma^n (V/r)$$

The total internal resistance is the sum of $r_1, r_2, r_3 \ldots r_n$ in parallel (called hereafter $\Sigma^n r_{(\text{parallel})}$) and it follows that any number of batteries are equivalent in effect to a single unit having an E.M.F.

$$\Sigma^n (V/r) \times \Sigma^n r_{\text{ (parallel)}}$$

and an internal resistance $\Sigma^n r_{\text{ (parallel)}}$. The power on short-circuit is therefore

$$(\Sigma^n(V/r))^2 \times \Sigma^n r_{\text{ (parallel)}} \qquad \ldots \quad (8)$$

It will be convenient before passing on to the case of valves in parallel to obtain an expression for the equivalent resistance external to any element of the network of

Fig. 1. The total resistance external to any E.M.F. V_n is

$$\frac{V_n}{i_n} - r_n.$$

If $n = 2$ we have (from equation (6))—

Resistance external to V_1

$$= \frac{V_1}{i_1} - r_1$$

$$= \frac{V_1(r_1R_e + r_2R_e + r_1r_2)}{V_1R_e + V_1r_2 - V_2R_e} - r_1$$

$$= R_e \frac{(V_1r_2 + V_2r_1)}{V_1r_2 + V_1R_e - V_2R_e} \quad \ldots \quad (9)$$

If $V_1 = V_2$ and $r_1 = r_2$ this becomes $2R_e$ which is seen to be correct as the external current is shared equally between the batteries.

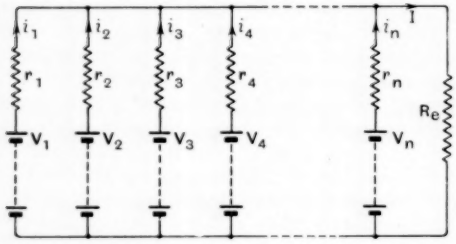


Fig. 1.

Again, if

$$R_e = \frac{V_2}{V_1 - V_2} \cdot r_1$$

so that $i_2 = 0$, equation (9) reduces to R_e.

The apparent external resistance, looking into the circuit from V_2, can similarly be shown to be—

Resistance external to V_2

$$= R_e \frac{V_1r_2 + V_2r_1}{V_2r_1 - V_1R_e + V_2R_e} \quad \ldots \quad (10)$$

Equations (9) and (10) warn us to beware of assuming that all the valves in one bank are necessarily working into the same effective load.

2. Application to Case of n Valves in Parallel.

We are now in a position to consider the network of Fig. 2, where the batteries are replaced by valves having unilateral conductivity. Since we are only concerned with A.C. values we may regard the valves as alternators developing E.M.F.s of maximum value $\mu_1 \mu_2, \ldots \mu_n$ times the common applied grid voltage δv_g. If we assume in the first instance that the region of linear operation is unlimited in the positive direction, and continues without modification down to zero anode current, we may

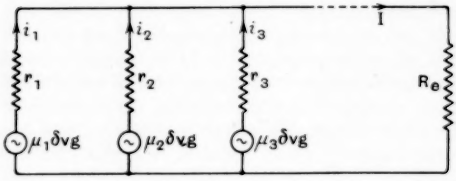


Fig. 2.

note first that although the total current always increases with the applied grid voltage (equations (5), (6), (7)), there may be one or more valves in which the anode current falls as the total current rises, this effect depending upon the magnitude of the external resistance R_e. This condition is illustrated by actual curves in Fig. 3, which shows the resulting anode current in the extreme case of an LS5 and an LS5B valve operating in parallel with a common external resistance of 20,000 ohms. Curves A and B refer to the LS5 and LS5B valves separately and represent the anode current

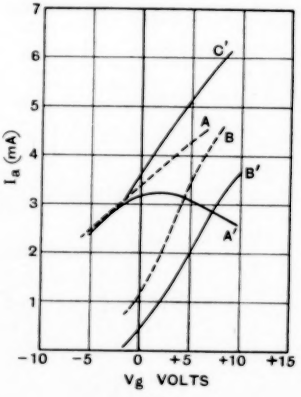


Fig. 3.

at 120 volts with the above value of external resistance. Curves A' and B' represent the separated anode currents for the same two valves when operating in parallel, while Curve C' gives the resulting anode current

through the resistance. It is seen that over a certain portion of the characteristics, where μ and r have appropriate values, the anode current change for the LS5 is zero or negative, although the total change of anode current is always positive.

The following example will serve to show that this result is not necessarily restricted to those cases in which the valve characteristics are intentionally dissimilar. From half-a-dozen valves of the same (LS5A) type, two were chosen as having the following values of amplification factor and anode A.C. resistance :—

	Amplification factor.	Anode A.C. resistance.
No. 1 ...	$\mu_1=2$	$r_1=3{,}000$ ohms.
No. 2 ...	$\mu_2=2.19$	$r_2=3{,}350$ ohms.

Now it is shown graphically in an important article by E. Green* on "The Use of Plate-Current Plate-Voltage Characteristics" that the external resistive load can profitably be made 11.5 times the anode A.C. resistance of the type of valve we are considering, *i.e.*, with two perfectly matched valves of 3,000 ohms each in parallel, R_e may be $11.5\times1{,}500=17{,}200$ ohms if V_a is in the neighbourhood of 500 volts. It would appear, however, from the above result that the use of so high an external resistance in association with valves placed in parallel (without extremely careful selection) would lead to serious losses.

If the reader cares to compare the theoretical short-circuited outputs for a few selected cases by means of equation (8), he

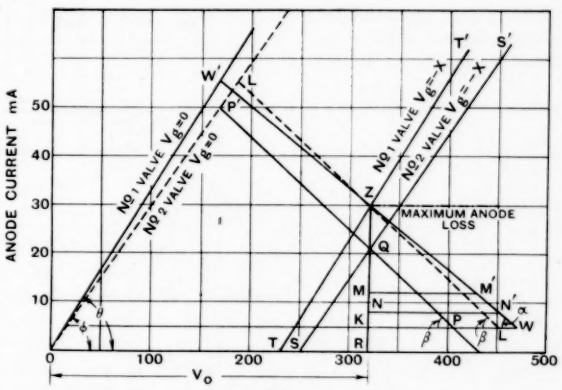


Fig. 4.

The difference between μ_1 and μ_2 is 10 per cent. while μ_1/r_1 and μ_2/r_2 are nearly equal. If these two valves are paralleled and connected to a common resistive load R_e, equation (6) shows that if

$$R_e=\frac{\mu_1}{\mu_2-\mu_1}\times r_2=35{,}000 \text{ ohms approx.}$$

i_1 will become vanishingly small (zero with linear characteristics), while if R_e exceeds this value, i_1 will be "negative" (*i.e.*, 180 degrees out of phase with i_2).

It can also be shown by means of equations (5) and (6) that No. 1 valve will only be contributing one-third of the total current I when

$$\frac{i_1}{I}=\frac{1}{3}=\frac{\mu_1R_e+\mu_1r_2-\mu_2R_e}{\mu_1r_2+\mu_2r_1}$$

i.e., when $R_e=12{,}000$ ohms approx.

will find that even here there is a loss of power resulting from a combination consisting of $n/2$ valves having μ and r values (say) 10 per cent. above, and $n/2$ valves—having values 10 per cent. below some nominal value. Were the characteristics linear this loss would not be of much importance, amounting to about 5 per cent. in a typical case.† But as the constants of the valves are made to differ more widely, the currents i_1 and i_2 (equations (6) and (7)) diverge more and more above and below equality. That is to say, that of two valves in which $\mu_1>\mu_2$ and $r_1>r_2$, the valve with

* *E.W. & W.E.*, July-August, 1926. The ensuing discussion is based on the methods therein described.

† The percentage loss is independent of n.

the higher amplification factor μ_1 is called upon to deliver a larger current than the other valve, which is therefore working inefficiently. Moreover, if the grids are held at a common negative potential, this potential must be chosen so that the valve with lowest amplification factor is limited to some definite anode loss (about 10 watts in the present case).

Let us see how this affects the conditions of mutual operation. Returning to the numerical example with which we began this section, Fig. 4 shows a portion of the I_a—V_a characteristics for the two valves. No D.C. loss is assumed in the feed circuits and the valves are working into resistive loads $R_{e1}=cot\,\alpha$ and $R_{e2}=cot\,\beta$. These loads are obtained from equations (9) and (10) and represent for each valve an arbitrary external load of R_e=2,800 ohms, as modified by the presence of the other valve. Thus :—

Load external to No. 1 valve

$$= cot\,\alpha = R_{e1}$$

$$= R_e\frac{(\mu_1 r_2+\mu_2 r_1)}{\mu_1 R_e+\mu_1 r_2-\mu_2 R_e}$$

$$= 2{,}800\,\frac{(2\times 3{,}350+2.19\times 3{,}000)}{(2\times 2{,}800+2\times 3{,}750-2.19\times 2{,}800)}$$

$$= 2{,}800\times 2.15$$

$$= 6{,}000 \text{ ohms.}$$

Load external to No. 2 valve

$$= cot\,\beta = R_{e2}$$

$$= R_e\frac{(\mu_1 r_2+\mu_2 r_1)}{\mu_2 r_1-\mu_1 R_e+\mu_2 R_e}$$

$$= 2{,}800\,\frac{13{,}270}{2.19\times 300-2\times 2{,}800+2.19\times 2{,}800}$$

$$= 5{,}300 \text{ ohms.}$$

If TT' is the line of constant grid voltage ($-x$) on the set of characteristics for No. 1 valve, and SS' represents the same voltage for No. 2 valve, we have

$$\frac{OT}{\mu_1}=\frac{OS}{\mu_2} \qquad \ldots \qquad \ldots (11)$$

Since both valves must be restricted to a certain anode dissipation, assumed in this case to be 0.03A at 320 volts, ZR = 0.03A is the steady current for No. 1 valve. Also, under the conditions enumerated, the steady current of No. 2 valve is seen to be QR= 0.021A,* while the maximum distortionless power output† from this valve (taking 5mA as the minimum anode current) is :—

$$\tfrac{1}{2}.QK.KP = \tfrac{1}{2}\times 0.016\times 85 = 0.68 \text{ watt.}$$

We can determine the excursion of anode current for No. 1 valve corresponding to the excursion QK for No. 2 by means of equations (6) and (7).

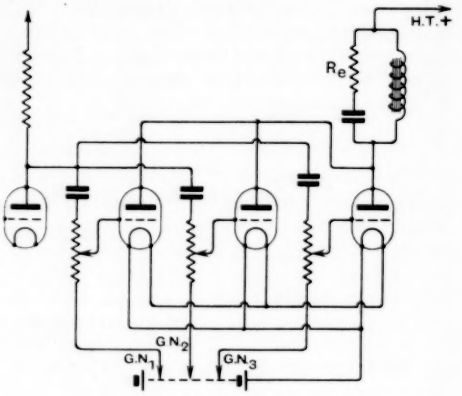


Fig. 5.

Thus

$$\frac{i_1}{i_2}=\frac{\mu_1 R_e+\mu_1 r_2-\mu_2 R_e}{\mu_2 R_e-\mu_2 r_1-\mu_1 R_e}$$

where R_e=2,800 ohms.

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* The value of the steady current i_2 in No. 2 valve relative to the current i_1 in No. 1 can be obtained for the general case as follows :—

We have, from equation (11)

$$OS=\frac{\mu_2}{\mu_1}OT$$

$\therefore$ $TS=\psi\,OT$, where $\psi=\dfrac{\mu_2}{\mu_1}-1$

$\therefore$ $TS=\psi(OR-TR)=\psi(V_0-i_1\,cot\,\theta)$

also $SR=TR-TS$

$$=i_1\,cot\,\theta-\psi V_0+\psi i_1\,cot\,\theta$$

$\therefore$ $QR=i_2=SR\,tan\,\phi$

$$=i_1\,cot\,\theta\,tan\,\phi-\psi V_0\,tan\,\phi+\psi i_1\,cot\,\theta\,tan\,\phi.$$

But $cot\,\theta=r_1$ and $tan\,\phi=1/r_2$

$\therefore$ $$i_2=i_1\left(\frac{r_1}{r_2}+\psi\frac{r_1}{r_2}\right)-\psi\frac{V_0}{r_2}$$

$$=\frac{\mu_2 r_1}{\mu_1 r_2}i_1-\frac{V_0}{r_2}\left(\frac{\mu_2}{\mu_1}-1\right)$$

If $\mu_2>\mu_1$ and $\mu_1/r_1=\mu_2/r_2$, it is seen that i_2 is always less than i_1.

† By distortionless output it is meant that grid current and bottom-bend working are excluded.

$$\therefore \quad i_1 = i_2 \times \frac{2 \times 2{,}800 + 2 \times 3{,}350 - 2.19 \times 2{,}800}{2.19 \times 2{,}800 + 2.19 \times 3{,}000 - 2 \times 2{,}800}$$

$$= 0.016 \times \frac{6{,}168}{7{,}100} \quad \ldots \quad \ldots \quad \ldots \quad (12)$$

$$= 0.018\text{A} = ZM \text{ (Fig. 4).}$$

So that the worst-placed valve (No. 2) controls the situation, and No. 1, although potentially capable of handling larger input, cannot receive this without No. 2 becoming overloaded. The total power output is 0.68 watt from No. 2 valve and $\frac{1}{2} . ZM . MM' = \frac{1}{2} \times 0.018 \times 84.5 = 0.76$ watt from No. 1, or 1.44 watts in all.

It is evident that the use of separate grid bias adjustments will enable us to obtain a much better output. This is usually provided in transmitting circuits but would seem to be quite desirable also in receivers (power amplifiers) unless facilities exist for the selection of a suitable team of valves for parallel operation.

If, then, we arrange separate grid bias, No. 2 valve can be independently adjusted for about 10 watts anode dissipation, its working line then being LL' and its maximum output

$\frac{1}{2}.ZK.KL = \frac{1}{2} \times 0.025 \times 130 = 1.62$ watts.

The anode current excursion for No. 1 valve is, by equation (12)

$$i_1 = i_2 \times \frac{6{,}618}{7{,}100}$$

$$= 0.022\text{A} = ZN$$

and the power output is

$$\frac{1}{2} . ZN . NN' = \frac{1}{2} \times 0.022 \times 130$$

$$= 1.43 \text{ watts.}$$

The total power output is now therefore $1.62 + 1.43 = 3.05$ watts, or more than twice the value obtained with common grid bias.

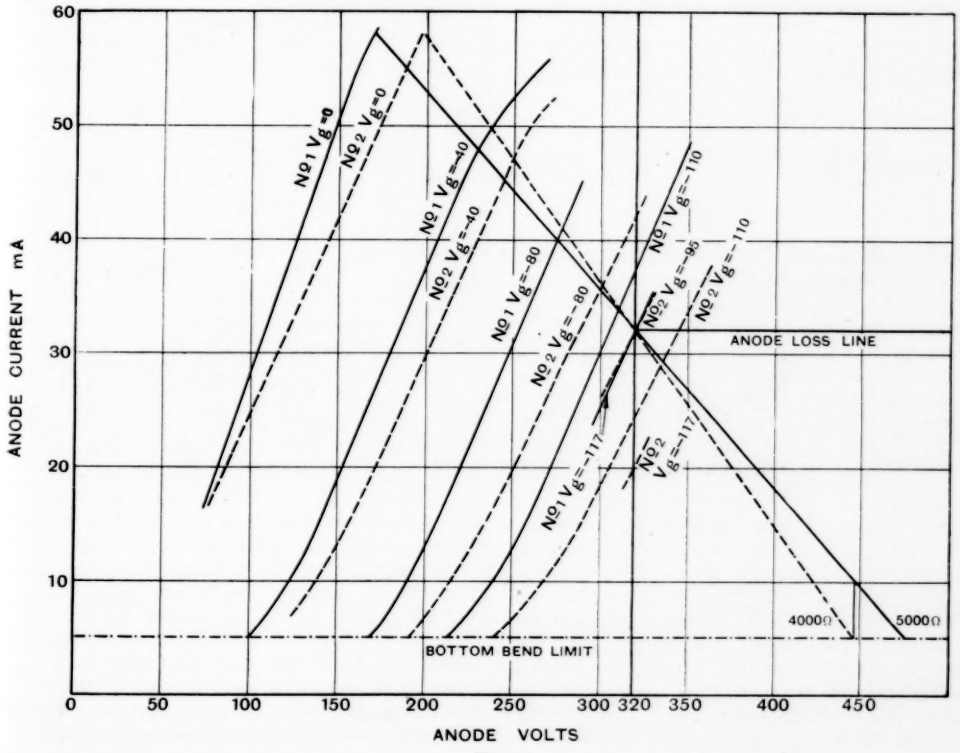


Fig. 6.

In order to carry the operating points of all the valves in any given bank to their extreme limits of distortionless working it would be necessary to apply to each grid a certain fraction of the alternating voltage developed by the previous valve. This could be adjusted by means of potentiometers in place of grid-leaks, as shown in Fig. 5. With transformer coupling a tapped secondary or a tapped resistance across the secondary would be used. In the case of the two valves given in the above example, the increase in output obtained by applying graded input voltages in addition to separate grid bias adjustments would be from $1.62 + 1.42 = 3.04$ watts to $1.62 + 1,875 = 3.5$ watts, or an increase of 15 per cent.

In conclusion, we may glance briefly at a further example in which the characteristics for two other LS5A valves have been observationally plotted (see Fig. 6). The constants in this case are as follows :—

	Amplification factor.	Anode A.C. resistance.
No. 1 ...	$\mu_1 = 1.76$	$r_1 = 2,800$ ohms.
No. 2 ...	$\mu_2 = 2$	$r_2 = 3,100$ ohms.

As before, we find that

$$\frac{\mu_1}{r_1} \doteq \frac{\mu_2}{r_2}\text{; also } \frac{\mu_2}{\mu_1} = 114 \text{ per cent.}$$

and

$$\frac{r_2}{r_1} = 110 \text{ per cent.}$$

In this case we shall at once apply separate grid bias, and we find that for No. 1 valve it must be —117 volts and for No. 2 —95 volts to limit the static dissipation in each valve to 10 watts, viz., 32mA at 320 volts. If the actual external load is 2,200 ohms, the apparent load on No. 1 valve comes out at 5,000 ohms and that on No. 2 at 4,000 ohms. Allowing a minimum anode current of 5mA it will be seen on reference to Fig. 6 that the maximum power from No. 2 valve is roughly

$$\tfrac{1}{2} \times \frac{25 \times 120}{1,000} \text{ watts} = 1.5 \text{ watts.}$$

The maximum power from No. 1 valve with common alternating grid voltage is

$$\tfrac{1}{2} \times \frac{21 \times 120}{1,000} = 1.25 \text{ watts,}$$

making a total of 2.76 watts. If, however, the input to No. 1 is separately applied, and increased to the maximum permissible value for this valve, the available power output is

$$\tfrac{1}{2} \times \frac{25 \times 150}{1,000} = 1.88 \text{ watts.}$$

giving a total of 3.38 watts. The improvement factor is therefore

$$\left(\frac{3.38}{2.76} - 1\right) \times 100 \text{ per cent.} = 22 \text{ per cent.}$$

Finally, with a bank of say five valves similar to No. 1 and one similar to No. 2, it is evident that the improvement factor would be considerably in excess of this figure.

It should be noted, however, that the percentage of second harmonics introduced by one *abnormal* valve is inversely proportional to the number of *normal* valves (with non-distorting adjustments) in parallel. Despite the numerical examples, therefore, it may be doubted whether the application to Modulators or Power Amplifiers of the method outlined above would give rise to very well-marked improvements. There remains also the fact that besides compensating valves for their varying "μ" values, their A.C. resistances ought also theoretically to be compensated by means of separate output transformers. Possibly manufacturers would solve the difficulty by undertaking to supply small groups of power valves having characteristics guaranteed to fall within close limits.

Resonance in Series and Parallel Circuits.

By H. J. Boyland, A.M.I.E.E.

IT would appear rather unfortunate that the term " resonance " was ever employed to describe certain electrical phenomena, inasmuch as its precise meaning when so used is by no means well defined, and the so-called conditions of resonance depend upon what interpretation is placed upon the term. It was first shown by Kelvin that the discharge of a condenser through a circuit containing inductance is of an oscillatory nature, provided that the resistance included in the circuit be below a certain limit. The frequency of this oscillatory discharge of the condenser is known as the " natural " or " free " oscillation frequency of the circuit, and, strictly speaking, resonance is the condition which exists when a sinusoidal E.M.F. of frequency equal to the natural frequency is applied to the circuit, and this is so whether the E.M.F. be applied to the condenser and inductance in series or in parallel. With the condenser and inductance in series the current which flows through the combination from the external source is limited only by the resistance of the circuit, and the potential differences across the inductance and across the condenser attain abnormal values, many times in excess of the applied E.M.F. The combination of an inductance and condenser in parallel, however, offers to an applied E.M.F. of frequency equal to the natural frequency an impedance of very high value, which results in the current flowing from the source being very small ; moreover, in this case a large oscillating current surges round the closed circuit. If, however, the resistance be not negligible these effects do not have their maxima when the applied frequency is equal to the natural frequency and hence the above general definition of resonance ceases to be of any particular significance.

The object of this article is to analyse certain combinations of inductance, capacity and resistance in order to derive the relationship which must exist between these quantities to satisfy the various conditions set out below :—

For Series Circuits.

(*a*) Potential difference across inductance to be a maximum.
(*b*) Potential difference across capacity to be a maximum.
(*c*) Current through circuit to be a maximum.

For Parallel Circuits.

(*d*) Equivalent reactance of circuit to be zero.
(*e*) Impedance to be a maximum.

We shall consider the following circuits :—

Series Circuits.

1. Inductance and capacity in series without resistance.
2. Inductance, capacity, and resistance in series.
3. Inductance with included resistance in series with capacity.

Parallel Circuits.

4. Pure inductance (*i.e.*, without resistance) and pure capacity in parallel.
5. Inductance and resistance in parallel with pure capacity.
6. Inductance and resistance in parallel with capacity and resistance.

In every case we shall assume an applied sinusoidal E.M.F., *i.e.*, an alternating E.M.F. of pure sine wave form without harmonics, and that all inductances and capacities are concentrated. While it would be sufficient to deal with the most general case of each arrangement (series and parallel) it is felt that it will be more instructive to treat each case separately starting with the simplest. In general it will be found that the relationship between the constants of a circuit to satisfy any given condition will be different according to the nature of the variable of the circuit. For example, with inductance and resistance in parallel with pure capacity,

the relationship which holds between the quantities for maximum impedance of the combination is different according to whether the inductance, capacity or frequency is varied in order to obtain this condition. Hence in most cases it is necessary to consider separately the effect of varying the inductance, the capacity and the frequency. Since $2\pi f=\omega$, where f is the frequency, we shall consider the effect of varying ω instead of f in order to avoid the constant repetition of 2π.

The following symbols will be used :—

I=Current in amperes.
L=Inductance in henries.
R=Resistance in ohms.
b=Susceptance in mhos.
E=Applied E.M.F. in volts.
C=Capacity in farads.
g=Conductance in mhos.
V=Potential difference in volts.

Series Circuits.

The current which flows through a circuit containing inductance, capacity and resistance in series is given by the expression

$$I=\frac{E}{\sqrt{R^2+\left(\omega L-\frac{1}{\omega C}\right)^2}}$$

R being the total resistance of the circuit, however this resistance may be distributed.

Inductance and Capacity in Series without Resistance.

Since the resistance of the circuit is zero the expression for the current reduces to

$$I=\frac{E}{\omega L-\frac{1}{\omega C}}$$

From inspection of this equation it is obvious that the current will be a maximum (in this case infinitely large) when $\omega L=1/\omega C$, *i.e.*, when the inductive reactance is equal to the capacity reactance, or when $\omega=1/\sqrt{LC}$ (condition *c*). Also since the potential difference across the inductance is given by $V_L=\omega LI$, and if I is a maximum, it can be shown that V_L will also be a maximum when $\omega=1/\sqrt{LC}$ (condition *a*).

The potential difference across the condenser is given by $I/\omega C$ and this also is a maximum when I is a maximum, *i.e.*, when $\omega L=1/\omega C$ (condition *b*). The natural frequency of oscillation of a closed circuit can be obtained from the expression

$$2\pi f=\omega=\sqrt{\frac{1}{LC}-\frac{R^2}{4L^2}}$$

and if R be negligible this reduces to the form $\omega=1/\sqrt{LC}$; ω in this case being $2\pi\times$natural frequency of oscillation. Hence we see that, with a simple series circuit of negligible resistance, when the applied frequency is equal to the natural frequency we have maximum current, maximum potential difference across the inductance, and maximum potential difference across the condenser all occurring simultaneously. We will now investigate the effect of resistance.

Inductance, Capacity and Resistance in Series.

The potential difference across the inductance is given by $V_L=\omega LI$ and substituting the value of I in this expression we obtain

$$V_L=\frac{\omega LE}{\sqrt{R^2+\left(\omega L-\frac{1}{\omega C}\right)^2}}$$

To satisfy condition *a* this must be a maximum and, as pointed out above, we have to consider separately the effect of inductance, capacity, and frequency variations.

Inductance Variation.

To find the required relationship the simplest method is to differentiate the above expression for V_L with respect to L and equate to zero.

$$V_L^2=\frac{\omega^2L^2E^2}{R^2+\left(\omega L-\frac{1}{\omega C}\right)^2}$$

differentiating gives

$$2V_L\frac{dV_L}{dL}=\frac{-2\omega^3L^2E\left(\omega L-\frac{1}{\omega C}\right)}{\left[R^2+\left(\omega L-\frac{1}{\omega C}\right)^2\right]^2}$$

$$2\omega^2LE^2\left[R^2+\left(\omega L-\frac{1}{\omega C}\right)^2\right]$$

equating to zero and simplifying we obtain

$$\omega = \frac{1}{\sqrt{LC - R^2C^2}} \quad .. \quad (1)$$

i.e., when $L = \frac{1}{\omega^2 C} + R^2C$

the potential difference across the inductance will be a maximum.

Capacity Variation.

Differentiating the equation for V_L^2 with respect to C, we obtain—

$$2V_L \frac{dV_L}{dC} = \frac{-2\omega^2L^2E^2\left(\omega L - \frac{1}{\omega C}\right)\left(\frac{1}{\omega C^2}\right)}{\left[R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2\right]^2}$$

Equating to zero and simplifying gives

$$\omega = \frac{1}{\sqrt{LC}} \quad .. \quad .. \quad (2)$$

Hence when $C = 1/\omega^2 L$ the potential difference across the inductance will be a maximum.

Frequency Variation.

Differentiating the equation for V_L^2 with respect to ω we obtain—

$$2V_L \frac{dV_L}{d\omega} = \frac{-2\left(\omega L - \frac{1}{\omega C}\right)\left(L + \frac{1}{\omega^2 C}\right)\omega^2L^2E^2}{\left[R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2\right]^2}$$

$$2\omega L^2E^2\left[R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2\right]$$

equating to zero gives—

$$R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2 = \left(\omega L - \frac{1}{\omega C}\right)\left(\omega L + \frac{1}{\omega C}\right)$$

from which

$$\omega = \frac{1}{\sqrt{LC - \frac{C^2R^2}{2}}} \quad .. \quad (3)$$

and this value of ω will be found to make V_L a maximum.

To satisfy condition *b*, *i.e.*, to obtain maximum potential difference across the capacity, we next must proceed in exactly the same way as before. The potential difference at the terminals of a condenser is given by the expression $V_C = I/\omega C$ and substituting the value of I we obtain

$$V_C = \frac{E}{\omega C\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

and

$$V_C^2 = \frac{E^2}{\omega^2C^2\left[R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2\right]}$$

Inductance Variation.

Differentiating with respect to L gives

$$2V_C \frac{dV_C}{dL} = \frac{-E^2(2\omega^4C^2L - 2\omega^2C)}{\omega^4C^4\left[R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2\right]^2}$$

Equating to zero and solving we get

$$\omega = \frac{1}{\sqrt{LC}} \quad .. \quad (4)$$

i.e., the potential difference at the terminals of the condenser is greatest when $L = 1/\omega^2 C$.

Capacity Variation.

Differentiating with respect to C gives

$$2V_C \frac{dV_C}{dC} = \frac{-E^2(2\omega^2CR^2 + 2\omega^4L^2C - 2L\omega^2)}{\omega^4C^4\left[R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2\right]^2}$$

Equating to zero and solving we get

$$\omega = \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}} \quad .. \quad (5)$$

i.e., when $C = \frac{1}{\omega^2C^2 + R^2}$

the potential difference at the terminals of the condenser is greatest.

Frequency Variation.

Differentiating with respect to ω gives

$$2V_C \frac{dV_C}{d\omega} = \frac{-E^2(2\omega C^2R^2 + 4\omega^3L^2C^2 - 4LC\omega)}{\omega^4C^4\left[R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2\right]^2}$$

Equating to zero and solving we get

$$\omega = \sqrt{\frac{1}{LC} - \frac{R^2}{2L^2}} \quad .. \quad (6)$$

and this value for ω will be found to give the maximum potential difference at the terminals of the condenser.

To satisfy condition *c*, *i.e.*, to obtain maximum current, it is obvious that the expression for the impedance, *i.e.*,

$$\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

must be a minimum; since R is constant this expression will be a minimum when

$$\omega L = 1/\omega C \text{ or when } \omega = \frac{1}{\sqrt{LC}} \qquad (7)$$

and this will be so whether we vary L, C or ω.

Inductance with Included Resistance in Series with Capacity.

Since in this case the inductance has a resistance R, the potential difference at its terminals when a current I flows through it is given by

$$V_L = I\sqrt{R^2 + \omega^2 L^2}$$

and substituting the value of I gives

$$V_L = \frac{E\sqrt{R^2 + \omega^2 L^2}}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

or

$$V_L^2 = \frac{E^2(R^2 + \omega^2 L^2)}{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

and to satisfy condition *a*, V_L must be a maximum.

Inductance Variation.

Differentiating with respect to L gives

$$2V_L \frac{dV_L}{dL} = \frac{-2E^2R^2\left(\omega L - \frac{1}{\omega C}\right)\omega + 2E^2\omega^2 L\left[R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2\right] - 2\omega^3 E^2 L^2\left(\omega L - \frac{1}{\omega C}\right)}{\left[R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2\right]^2}$$

Equating to zero we have

$$\omega^2 L\left[R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2\right] = \omega R^2\left(\omega L - \frac{1}{\omega C}\right) + L^2\omega^3\left(\omega L - \frac{1}{\omega C}\right)$$

from which $\omega^2 CL^2 - L - R^2 C = 0$,

and hence

$$\omega = \sqrt{\frac{1}{LC} + \frac{R^2}{L^2}} \qquad (8)$$

i.e., the potential difference at the terminals of the inductance has a maximum value when

$$L = \frac{1 + \sqrt{1 + 4R^2C^2\omega^2}}{2\omega^2 C}$$

Capacity Variation.

Differentiating with respect to C gives

$$2V_L \frac{dV_L}{dC} = \frac{-2\left(\omega L - \frac{1}{\omega C}\right)\left(\frac{1}{\omega C^2}\right)}{\left[R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2\right]^2}$$

Equating to zero and solving we have,

$$\omega = \frac{1}{\sqrt{LC}} \qquad (9)$$

Thus it will be found that V_L will be a maximum when $C = 1/\omega L^2$.

Frequency Variation.

Differentiating with respect to ω gives

$$2V_L \frac{dV_L}{d\omega} = \frac{-2E^2R^2\left(\omega L - \frac{1}{\omega C}\right)\left(L + \frac{1}{\omega^2 C}\right) + 2E^2\omega L^2\left[R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2\right] - 2E^2\omega^2 L^2\left(\omega L - \frac{1}{\omega C}\right)\left(L + \frac{1}{\omega^2 C}\right)}{\left[R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2\right]^2}$$

Equating to zero we have

$$\omega L^2\left[R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2\right] = R^2\left(\omega L - \frac{1}{\omega C}\right)\left(L + \frac{1}{\omega^2 C}\right) + \omega^2 L^2\left(\omega L - \frac{1}{\omega C}\right)\left(L + \frac{1}{\omega^2 C}\right)$$

which when simplified becomes—

$$2\omega^4 L^3 C - 2\omega^2 L^2 - R^2 = 0$$

from which

$$\omega^2 = \frac{L + \sqrt{L^2 + 2CLR^2}}{2L^2 C},$$

since only the positive value for ω^2 is admissible, and therefore

$$\omega = \left[L + \frac{\sqrt{L^2 + 2CLR^2}}{2L^2 C}\right]^{\frac{1}{2}} \qquad (10).$$

The relationship which must hold between the quantities to satisfy conditions *b* and *c* are given by equations (4), (5), (6) and (7), since the fact that the resistance is now integral with the inductance does not in any way affect the equation for the potential difference at the terminals of the condenser; neither does it affect the expression for the current through the circuit.

The treatment of the parallel circuits is slightly more complex. To satisfy condition *d* we require the equivalent reactance of the circuit to be zero. By equivalent reactance is meant the reactance of the series circuit which is electrically equivalent to the parallel circuit. Strictly speaking no series circuit can be the exact equivalent of a parallel circuit in all respects, but for deducing certain results it is convenient to resolve the parallel circuit into an "equivalent" series circuit. For example, if we make the equivalent reactance zero, the parallel circuit exhibits only resistance, that is to say, any current which flows from the applied source through the combination is in phase with the E.M.F.

Parallel Circuits

The method of treatment will be more clearly understood if we consider the simplest parallel circuit.

Pure Inductance in Parallel with Pure Capacity.

In order to satisfy condition *d* it will first be necessary to resolve this circuit into an equivalent series circuit. In dealing with series circuits we add reactances; thus in a series circuit consisting of inductances L_1, L_2, L_3, and capacities C_1, C_2, the total reactance is

$$\omega L_1+\omega L_2+\omega L_3-\frac{1}{\omega C_1}-\frac{1}{\omega C_2}$$

capacity reactance being considered negative. If the circuit also contains resistances R_1, R_2, R_3, the total resistance of this series circuit is $R_1+R_2+R_3$. Thus we are enabled to solve the circuit. For parallel circuits we must deal with other quantities called susceptance and conductance. Susceptance is defined as being $-X/Z^2$ where X is reactance and Z is impedance. Due to the presence of the minus sign it will be seen that capacity-susceptance will be positive and inductive-susceptance negative. Conductance is defined as being R/Z^2. For our parallel circuit therefore we must determine the conductance and susceptance for each branch, and since conductances and susceptances may be added algebraically we determine the total conductance and total susceptance of the circuit by adding the separate conductances and susceptances. Let G be the total conductance and B the total susceptance of a parallel circuit, then the reactance of the equivalent series circuit can be shown to be $-B/(G^2+B^2)$, and the resistance of the equivalent series circuit $G/(G^2+B^2)$. Since we merely require the equivalent reactance to be zero, *i.e.*, we require $-B/(G^2+B^2)$ to be zero, it will be sufficient for our purpose to determine B, the total susceptance and equate it to zero.

The impedance Z_L of the branch containing the inductance

$$=\omega L,$$

$$\therefore \quad Z_L{}^2=\omega^2L^2.$$

The reactance X_C of the branch containing the inductance $=\omega L$,

$\therefore$ the susceptance b_L of this branch

$$=\frac{-X_L}{Z_L{}^2}=\frac{-\omega L}{\omega^2L^2}=-\frac{1}{\omega L}$$

The impedance Z_C of the branch containing the capacity

$$=\frac{1}{\omega C}$$

$$\therefore \quad Z_C{}^2=\frac{1}{\omega^2C^2}$$

The reactance X_C of the branch containing the capacity $=-1/\omega C$

$\therefore$ the susceptance b_C of this branch

$$=\frac{-X_C}{Z_C{}^2}=\frac{-(-1/\omega C)}{1/\omega^2C^2}=\omega C$$

The total susceptance therefore

$$=B=b_L+b_C=\omega C-1/\omega L,$$

and this is to be zero.

Hence the required condition is satisfied when

$$\omega C=1/\omega L,$$

or

$$\omega=\frac{1}{\sqrt{L\,C}} \qquad .. \quad (11)$$

and this result will obviously be obtained whichever of the quantities L, C, or ω be varied in order to satisfy the condition.

To satisfy condition *e* we require the impedance of the circuit to be a maximum. Now the impedance of the parallel circuit, considered as a whole, is given by the expression $1/(G^2+B^2)$. Also, since the resistance in each arm of the circuit is zero, the conductance of each arm, and therefore the total conductance, is zero; *i.e.*, $G=0$. When $\omega = 1/\sqrt{LC}$ we have seen that B is also zero, therefore it follows that when ω has this value the impedance of the circuit is infinitely great. Thus the required condition is satisfied when

$$\omega = 1/\sqrt{LC} \quad .. \quad .. \quad (12)$$

This circuit is, of course, that of the rejector type wave-trap.

Inductance and Resistance in Parallel with Pure Capacity.

Condition *d*, equivalent reactance to be zero.

The impedance Z_L of the branch containing the inductance

$$= \sqrt{R^2+\omega^2L^2} \quad \therefore \quad Z_L{}^2 = R^2+\omega^2L^2.$$

The reactance X_L of the branch containing the inductance $=\omega L$. Therefore the susceptance b_L of this branch

$$= \frac{-X_L}{Z_L{}^2} = -\frac{\omega L}{R^2+\omega^2L^2}$$

The impedance Z_C of the branch containing the capacity

$$= 1/\omega C \quad \therefore \quad Z_C{}^2 = 1/\omega^2C^2$$

The reactance X_C of the branch containing the capacity $=-1/\omega C$. Therefore the susceptance b_C of this branch

$$= \frac{-X_C}{Z_C{}^2} = \frac{-(-1/\omega C)}{1/\omega^2C^2} = \omega C$$

Hence the total susceptance

$$= B = \omega C - \frac{\omega L}{R^2+\omega^2L^2}$$

and equating this to zero we have

$$\omega C = \frac{\omega L}{R^2+\omega^2L^2}$$

from which the value of ω to satisfy the required condition is found to be

$$\omega = \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}} \quad .. \quad (13)$$

Solving this equation for L we find

$$L = \frac{1 \pm \sqrt{1-4\omega^2C^2R^2}}{2\omega^2C}$$

thus if $4\omega^2C^2R^2<1$, there will be two values of L which satisfy the required condition. Condition *e*, impedance to be a maximum.

Now the impedance is given by

$$\frac{1}{\sqrt{G^2+B^2}}$$

and we therefore require

$$\sqrt{G^2+B^2}$$

to be a minimum. This quantity,

$$\sqrt{G^2+B^2}$$

is called the admittance of the circuit and is usually denoted by Y. We have already derived an expression for B, and found it equal to

$$\omega C - \frac{\omega L}{R^2+\omega^2L^2}$$

We must now determine G.

The conductance g_L of the branch containing the inductance

$$= \frac{R}{Z_L{}^2} = \frac{R}{R^2+\omega^2L^2}$$

The conductance g_C of the branch containing the capacity $=0$, since the resistance in this branch $= 0$.

Therefore the total conductance

$$= G = \frac{R}{R^2+\omega^2L^2}$$

Hence the admittance

$$= Y = \sqrt{G^2+B^2}$$

$$= \sqrt{\left(\frac{R}{R^2+\omega^2L^2}\right)^2 + \left(\omega C - \frac{\omega L}{R^2+\omega^2L^2}\right)^2}$$

or $$Y^2 = \left(\frac{R}{R^2+\omega^2L^2}\right)^2 + \left(\omega C - \frac{\omega L}{R^2+\omega^2L^2}\right)^2$$

Inductance Variation.

Differentiating the above expression with respect to L gives—

$$2Y\frac{dY}{dL} = 2\left[\frac{R}{R^2+\omega^2L^2}\right]\left[\frac{-2R\omega^2L}{(R^2+\omega^2L^2)^2}\right] + 2\left[\omega C - \frac{\omega L}{R^2+\omega^2L^2}\right]\left[\frac{-\omega(R^2+\omega^2L^2)+2\omega^3L^2}{(R^2+\omega^2L^2)^2}\right]$$

$$= \frac{[2\omega C(R^2+\omega^2L^2)-2\omega L]}{[2\omega^3L^2-\omega(R^2+\omega^2L^2)]-4R^2\omega^2L} \over (R^2+\omega^2L^2)^3$$

Equating this to zero, we have—

$$2\omega^4L^2CR^2+2\omega^6L^4C-2\omega^4L^3-4\omega^2CR^4-2\omega^2LR^2=0$$

or $$\omega^4CL^4-\omega^2L^3-(LR^2+CR^4)=0$$

from which $$\omega^2=\frac{1}{LC}+\frac{R^2}{L^2}$$

and hence $$\omega=\sqrt{\frac{1}{LC}+\frac{R^2}{L^2}} \quad .. \quad (14)$$

Thus when

$$L=\frac{1+\sqrt{1+4R^2C^2\omega^2}}{2\omega^2C}$$

it will be found that the admittance is a minimum or the impedance a maximum.

Capacity Variation.

Differentiating the expression for Y^2 with respect to C gives—

$$2Y\frac{dY}{dC}=2\omega\left[\omega C-\frac{\omega L}{R^2+\omega^2L^2}\right]$$

equating to zero and simplifying we have—

$$C\omega^2L^2+CR^2-L=0$$

from which

$$\omega=\sqrt{\frac{1}{LC}-\frac{R^2}{L^2}} \quad .. \quad (15)$$

i.e., when

$$C=\frac{L}{\omega^2L^2+R^2}$$

the impedance is a maximum.

Frequency Variation.

Differentiating the expression for Y^2 with respect to ω gives—

$$2Y\frac{dY}{d\omega}=2\left[\frac{R}{R^2+\omega^2L^2}\right]\left[\frac{-2\omega L^2R}{(R^2+\omega^2L^2)^2}\right]+2\left[\omega C-\frac{\omega L}{R^2+\omega^2L^2}\right]\left[C-\frac{[L(R^2+\omega^2L^2)-2\omega^2L^3]}{[R^2+\omega^2L^2]^2}\right]$$

Equating to zero, and multiplying through by $(R^2+\omega^2L^2)^3$ we have—

$$[\omega C(R^2+\omega^2L^2)-\omega L]\,[C(R^2+\omega^2L^2)^2-L(R^2+\omega^2L^2)+2\omega^2L^3]-2R^2\omega L^2=0.$$

Multiplying out

$$\omega^6C^2L^6+3\omega^4L^4C^2R^2+3\omega^2C^2R^4L^2-2\omega^2CR^2L^3-\omega^2L^4+C^2R^6-2CR^4L-R^2L^2=0$$

i.e., $$(\omega^2L^2+R^2)\,(\omega^4C^2L^4+2\omega^2C^2R^2L^2+C^2R^4-L^2-2CLR^2)=0.$$

Hence, $$\omega^4C^2L^4+2\omega^2C^2R^2L^2+(C^2R^4-L^2-2CLR^2)=0$$

from which

$$\omega^2=\frac{-CR^2+\sqrt{L^2+2LCR^2}}{L^2C} \quad .. \quad (16)$$

If R be put equal to zero, the equation reduces to 12.

Inductance and Resistance in Parallel with Capacity and Resistance.

Condition *d*, equivalent reactance to be zero.

The susceptance of the branch containing the inductance

$$=\frac{-\omega L}{R_L^2+\omega^2L^2}$$

where R_L is the resistance in this branch.

The susceptance of the branch containing the capacity

$$=\frac{1/\omega C}{R_C^2+\frac{1}{\omega^2C^2}}$$

where R_C is the resistance in this branch.

Therefore, the total susceptance

$$=\frac{1/\omega C}{R_C^2+\frac{1}{\omega^2C^2}}-\frac{\omega L}{R_L^2+\omega^2L^2}$$

and this is to be zero, *i.e.*,

$$\frac{1/\omega C}{R_C^2+\frac{1}{\omega^2C^2}}=\frac{\omega L}{R_L^2+\omega^2L^2}$$

or $$\omega C(R_L^2+\omega^2L^2)=\omega L(\omega^2C^2R_C^2+1)$$

from which

$$\omega^2=\frac{L-CR_L^2}{CL^2-C^2LR_C^2} \quad .. \quad (17)$$

Equation 13 is obtained from this general expression by putting $R_c=0$. It is interesting to note that if the time constants of the two branches are equal and also the resistances [*i.e.* if $R_C=R_L=\sqrt{L/C}$] the reactance is independent of frequency and equal to zero.

Condition *e*, impedance to be a maximum.

The total conductance of this circuit

$$=\frac{R_L}{R_L^2+\omega^2L^2}+\frac{R_C}{R_C^2+\frac{1}{\omega^2C^2}}$$

hence,

$$Y^2=G^2+B^2=\left[\frac{R_L}{R_L^2+\omega^2L^2}+\frac{R_C}{R_C^2+\frac{1}{\omega^2C^2}}\right]^2 + \left[\frac{1/\omega C}{R_C^2+\frac{1}{\omega^2C^2}}-\frac{\omega L}{R_L^2+\omega^2L^2}\right]^2$$

Write

A for $R_L^2+\omega^2L^2$ and D for $R_C^2+\frac{1}{\omega^2C^2}$.

Then

$$Y^2=\left[\frac{R_L}{A}+\frac{R_C}{D}\right]^2+\left[\frac{1/\omega C}{D}-\frac{\omega L}{A}\right]^2$$

$$=\frac{1}{A}+\frac{1}{D}+\frac{2R_LR_C}{AD}-\frac{2L/C}{AD}$$

and this expression is to be a minimum.

Inductance Variation.

Differentiating the given expression with respect to L gives—

$$2Y\frac{dY}{dL}=-\frac{dA/dL}{A^2}-\frac{dD/dL}{D^2}-\frac{\left[2R_LR_CA\frac{dD}{dL}+2R_LR_CD\frac{dA}{dL}\right]}{A^2D^2}-\frac{\left[\frac{2AD}{C}-\frac{2LA}{C}\cdot\frac{dD}{dL}-\frac{2LD}{C}\cdot\frac{dA}{dL}\right]}{A^2D^2}$$

since D does not contain L, $\frac{dD}{dL}=0$.

Hence equating to zero and multiplying through by A^2D we have—

$$-\frac{dA}{dL}D-2R_LR_C\frac{dA}{dL}-\frac{2A}{C}+\frac{2L}{C}\frac{dA}{dL}=0$$

$$dA/dL=2\omega^2L$$

$$\therefore\quad 2\omega^2L\left(\frac{2L}{C}-2R_LR_C-R_C^2-\frac{1}{\omega^2C^2}\right)-\frac{R_L}{C}-\frac{\omega^2L^2}{C}=0$$

and simplifying

$$\omega^2(L^2C-2R_LR_CLC^2-LC^2R_C^2)=L+R_L^2C$$

from which

$$\omega^2=\frac{L+R_L^2C}{LC(L-2R_LR_CC-CR_C^2)} \qquad \ldots\ (18)$$

i.e., the real value of L which satisfies the above equation will be found to make the impedance a maximum.

Capacity Variation.

Differentiating the given expression with respect to C gives—

$$2Y\frac{dY}{dC}=-\frac{dA/dC}{A^2}-\frac{dD/dC}{D^2}-\frac{\left[2R_LR_CA\frac{dD}{dC}+2R_LR_CD\frac{dA}{dC}\right]}{A^2D^2}-\frac{\left[-\frac{2LAD}{C^2}-\frac{2LA}{C}\cdot\frac{dD}{dC}-\frac{2LD}{C}\cdot\frac{dA}{dC}\right]}{A^2D^2}$$

Since A does not contain C, $dA/dC=0$. Hence equating to zero and multiplying through by AD^2 we have :—

$$-\frac{dD}{dC}A-2R_LR_C\frac{dD}{dC}+\frac{2LD}{C^2}+\frac{2L}{C}\cdot\frac{dD}{dC}=0.$$

$$\frac{dD}{dC}=-\frac{2}{\omega^2C^3}$$

$$\therefore\quad \frac{2}{\omega^2C^3}\left(2R_LR_C+R_L^2+\omega^2L^2-\frac{2L}{C}\right)+\frac{2L}{C^2}\left(R_C^2+\frac{1}{\omega^2C^2}\right)=0.$$

Multiplying out and simplifying

$$\omega^2(L^2C+LR_C^2C^2)=L-2R_LR_CC-R_L^2C$$

from which

$$\omega^2=\frac{L-2R_LR_CC-R_L^2C}{L^2C+LR_C^2C^2} \qquad \ldots\ (19)$$

Frequency Variation.

The expression for the frequency which makes the impedance a maximum is rather clumsy but is given for the sake of completeness.

Differentiating the expression for Y^2 with respect to ω gives—

$$2Y\frac{dY}{d\omega}=-\frac{dA/d\omega}{A^2}-\frac{dD/d\omega}{D^2}-\frac{\left[2R_LR_C\frac{dA}{d\omega}D+2R_LR_C\frac{dD}{d\omega}A\right]}{A^2D^2}+\frac{\frac{2LA}{C}\cdot\frac{dD}{d\omega}+\frac{2LD}{C}\cdot\frac{dA}{d\omega}}{A^2D^2}$$

equating to zero and multiplying through by A^2D^2

$$\frac{dD}{d\omega}\left(\frac{2LA}{C}-2R_LR_CA-A^2\right)+$$
$$+\frac{dA}{d\omega}\left(\frac{2LD}{C}-2R_LR_CD-D^2\right)=0.$$

Now $\frac{dA}{d\omega}=2\omega L^2$ and $\frac{dD}{d\omega}=-\frac{2}{\omega^3C^2}$.

Substituting these values in the expression, multiplying out and simplifying, we have—

$$\omega^4L^2C^2(L^2-C^2R_C^4-2R_LR_C^3C^2+2LCR_C^2)$$
$$+2\omega^2L^2C^2(R_L^2-R_C^2)$$
$$+R_L^4C^2+2R_L^3R_CC^2-2LR_L^2C-L^2=0 \quad (20)$$

The value of ω can be found in any particular case, but the general expression above does not admit of further simplification.

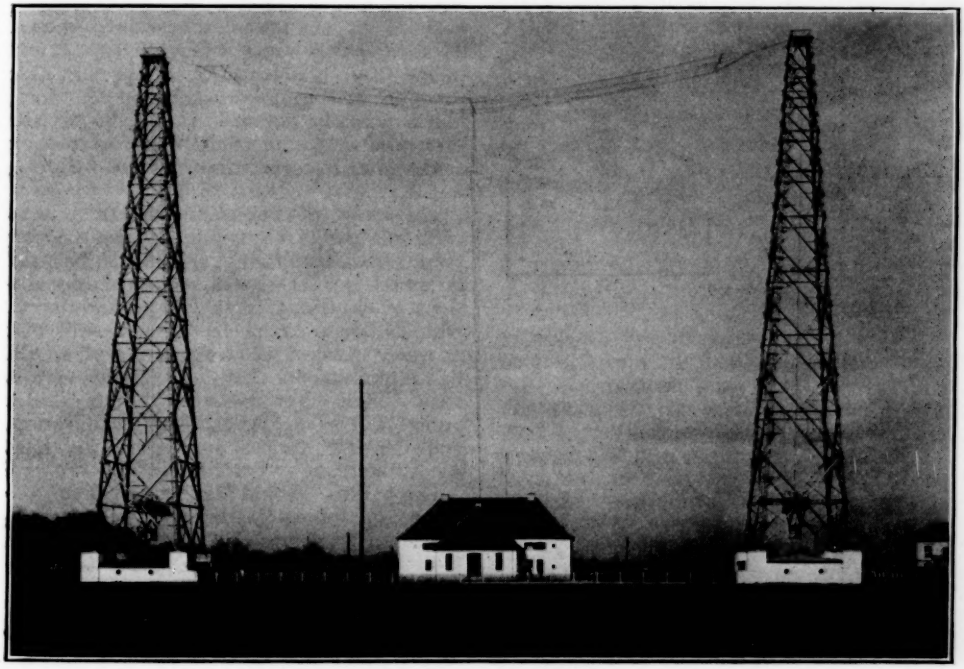
An interesting special case is when

$$R_L=R_C=\sqrt{L/C}.$$

With this value of resistance in each arm, the differential coefficient of the admittance with respect to ω is equal to zero. This means that the admittance, and consequently the current, is independent of the frequency.

It is hoped that the results deduced, and the method of analysis of these more elementary circuits will prove of use to those who are not already familiar with the subject.

Münich Broadcasting Station.



An interesting photograph which shows the wooden lattice aerial masts of the Münich Broadcasting Station, and the house which contains the transmitting apparatus and provides quarters for the permanent staff.

Mathematics for Wireless Amateurs.

By F. M. Colebrook, B.Sc., A.C.G.I., D.I.C.

(*Continued from page 612 of October issue.*)

PART IV.

APPLICATIONS TO ELECTRICAL PROBLEMS.

1. The Fundamental Laws of Current Networks.

THE whole theory of electric current networks, whatever be the nature of the conducting elements of the networks or of the currents flowing in them, is based on two remarkable generalisations known as Kirchhoff's first and second laws—remarkable because of their almost axiomatic simplicity and the wealth of information and deduction derivable from their application.

The first law—the algebraic sum of the currents which meet at any point in a network of conductors is zero.

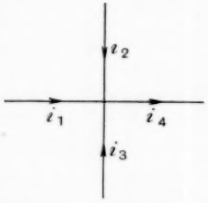


Fig. 36.

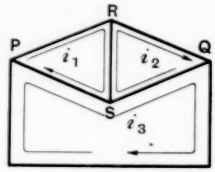


Fig. 37.

Notice the word "algebraic," which here, as always, means that sign must be taken into account. By "sign" is meant the sense of the current relative to the point considered. The usual convention in this matter is that a current will be reckoned positive if it is flowing towards the point, and negative if it is flowing away from the point. Thus the application of the law to the element of a network shown in Fig. 36 leads to the equation—

$$i_1 + i_2 + i_3 - i_4 = 0.$$

"But," says the reader, "how can I tell, in the case of a really complicated network, which way the currents are flowing?" The answer is: "You can't, but it doesn't matter." Not that a special guardian angel has been detailed for this job, but that the combination of correct analysis with known data (such as the nature and disposition of the acting electromotive forces) will automatically confirm or correct the assigned directions. Thus, if in the above example the current i_2 is in fact flowing away from the point, its evaluation will lead to a negative number —10, for instance, showing that it is a current of magnitude 10 flowing in a direction opposite to that indicated.

An alternative and preferable manner of representing the flow of current in a network is that illustrated in Fig. 37, where the actual currents are regarded as due to the superposition of the circuital currents shown. Thus the current in PR is i_1, in RQ i_2, and in RS $(i_1 \sim i_2)$. This form of representation saves the writing of several current equations, for it actually assumes and embodies the first law. The sum of the currents meeting at the point R, for instance, is $i_1 - i_2 - (i_1 - i_2)$, *i.e.*, 0.

Physically, the law states that there is no accumulation of electricity at any meeting point of conductors in a network. In point of actual fact, there will be local accumulations of electricity for an exceedingly short period after a circuit has been closed, just as water released into a system of pipes will first fill up the pipes before settling down to a steady flow, but in general this initial period will be negligibly short in duration, and the law applies exactly to the final steady state.

Kirchhoff's second law relates to the potential differences in a closed circuit. The term "potential difference" includes both "electromotive force," *i.e.*, a chemically or mechanically maintained potential difference which supplies energy to a circuit, and "back E.M.F." or fall of potential due to the passage of a current in a conductor. Consider, for instance, the passage of a current of magnitude i through a resistance R ohms, illustrated in Fig. 38. By Ohm's

law, a knowledge of which is assumed, the magnitude of the potential difference between the points a and b is iR volts, and the current flows "downhill," as one would expect it to. That is, a is at a higher potential than b. If the conductor R were removed and the terminals a and b maintained at the same potentials as before, a current would obviously tend to flow round the rest of the circuit in the direction shown by the dotted line, *i.e.*, in a direction opposite

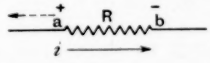


Fig. 38.

to that of the original current. Thus, if the direction of the current through the resistance be taken as positive, the appropriate sign to attribute to the potential difference iR is negative. Conversely, any potential difference in the closed circuit containing R which would tend to maintain the current in the same—*i.e.*, positive—direction can reasonably be given a positive sign. Allocating signs in this manner, one can form the algebraic sum of all the potential differences in any closed circuit, and Kirchhoff's second law states that this sum is zero. There should be no difficulty in appreciating the physical significance of the second law, for it means no more than this—that a man who sets out from his home on a roundabout journey up hill and down dale, and then comes home again, must of necessity in the course of his wanderings have gone uphill

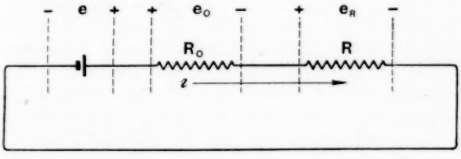


Fig. 39.

and downhill to exactly the same extent, since he has finished up at the level from which he started.

Applying the law by way of illustration to the circuit shown in Fig. 39, which represents a battery of E.M.F. e volts and internal resistance R_0 ohms supplying current to a resistance R ohms, we have—

$$e + e_0 + e_R = 0$$

i.e.,
$$e - iR_0 - iR = 0$$

or
$$i = e/(R_0 + R)$$

In the case of a varying current, *e.g.*, the high-frequency sine wave alternating current of wireless telegraphy, other "back E.M.F.s," or opposing potential differences, will come into play in addition to those due to the resistances of the conductors involved. It must be assumed that these ideas are already familiar to the reader, but a brief statement of them will be given for the sake of completeness.

2. Inductance.

A pure inductance opposes to a varying current i a back E.M.F. e_L proportional to the rate of change of the current, *i.e.*, proportional to di/dt. The definition of the unit of inductance is so chosen that the back E.M.F. in volts is $L(di/dt)$, L being the inductance in henries. A negative sign is attributed to it for the same reason as in the case of a pure resistance.

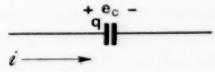


Fig. 40.

3. Capacity.

A pure capacity in a circuit carrying a varying current i (see Fig. 40) will oppose to the current a back E.M.F. proportional to the quantity of electricity stored in the condenser. The definition of the unit of capacity is so chosen that this back E.M.F. in volts is q/C, C being the capacity in farads, and q the quantity of electricity in coulombs stored on the positive electrode of the condenser, *i.e.*, leaving the matter of sign for the present—

$$|\, e_c \,| = q/C$$

Now, since the rate of change of q (*i.e.*, dq/dt) is the rate of flow of electricity along the conductor, *i.e.*, i amperes (or i coulombs per sec.), we have—

$$i = dq/dt$$

Therefore—

$$\frac{d\,|\,e_c\,|}{dt} = \frac{d}{dt}\left(\frac{q}{C}\right) = \frac{1}{C}\frac{dq}{dt} = \frac{i}{C}$$

Since the direction of the potential difference e_c relative to that of the current is such as to oppose the current, a negative sign is attributed to it in the above equation, giving—

$$\frac{de_c}{dt} = -\frac{i}{C}$$

4. Vectorial Representation of Back E.M.F.s.

It was shown in Para. 11, Part II (June, 1927), that a sine wave alternating current can be represented in the form

$$\mathbf{i}\cdot\boldsymbol{\nu} = \hat{\imath}\cos\omega t$$

where $\mathbf{i}$ is a vector of constant magnitude $\hat{\imath}$, rotating with constant angular velocity, and where $\boldsymbol{\nu}$ is a constant unit vector of

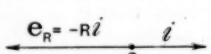


Fig. 41.

reference parallel to the bottom edge of the paper. (In the writer's opinion, this form of statement is preferable to the more usual description of the instantaneous value of the current as the "projection" on a given time axis of the rotating vector, since it permits the relationship between the vector and the current to be stated as an exact equation, as above).

(*a*) If such a current flows through a resistance R ohms, the back E.M.F. e_R is given, as already shown, by

$$e_R = -Ri$$

and since e_R is thus a simple multiple of i it will of necessity be a sine wave alternating potential difference of the same frequency as i, and can therefore be represented by a rotating vector $\mathbf{e}_R$ in the same manner. Expressing both the current and the back E.M.F. in vector form, we have

$$\mathbf{e}_R\cdot\boldsymbol{\nu} = -R(\mathbf{i}\cdot\boldsymbol{\nu}) = -R\mathbf{i}\cdot\boldsymbol{\nu}$$

This can be put in the form

$$(\mathbf{e}_R + R\mathbf{i})\cdot\boldsymbol{\nu} = 0$$

Since this is true at every instant, it follows that the vector $(\mathbf{e}_R + R\mathbf{i})$ is either zero at every instant or else is perpendicular to $\boldsymbol{\nu}$ at every instant. The second condition is obviously not fulfilled. Therefore

$$\mathbf{e}_R + R\mathbf{i} = 0 \quad \text{or} \quad \mathbf{e}_R = -R\mathbf{i}$$

This shows that the vector representing e_R is R times $\mathbf{i}$ in magnitude and opposite to it in direction, as in Fig. 41.

(*b*) For the back E.M.F. generated in a pure inductance L we have

$$\mathbf{e}_L = -L(di/dt)$$

and since the differential coefficient of a sine wave is a sine wave (or a cosine wave, which comes to the same thing) of the same frequency it follows that e_L can also be represented by a rotating vector of the same angular velocity as $\mathbf{i}$. Hence we have the scalar product equation

$$\mathbf{e}_L\cdot\boldsymbol{\nu} = -L\frac{d(\mathbf{i}\cdot\boldsymbol{\nu})}{dt}$$

Now it is easy to show (see Appendix I) that as $\boldsymbol{\nu}$ is a constant vector

$$\frac{d(\mathbf{i}\cdot\boldsymbol{\nu})}{dt} = \frac{d\mathbf{i}}{dt}\cdot\boldsymbol{\nu}$$

Further, it has been shown (see Para. 12, Part III, October, 1927) that for a vector of this character

$$d\mathbf{i}/dt = \omega j\mathbf{i}$$

Therefore

$$\mathbf{e}_L\cdot\boldsymbol{\nu} = -\omega jL\mathbf{i}\cdot\boldsymbol{\nu}$$

whence, as in case (*a*), $\mathbf{e}_L = -\omega jL\mathbf{i}$. The relation between the vectors $\mathbf{e}_L$ and $\mathbf{i}$ is therefore as shown in Fig. 42.

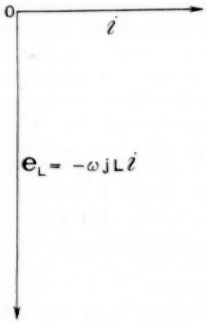


Fig. 42.

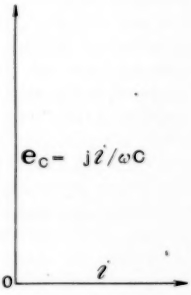


Fig. 43.

(*c*) With the above two examples the reader should have no difficulty in showing that the vector representing the back E.M.F. due to a condenser of capacity C is given by

$$\omega j\mathbf{e}_c = -\mathbf{i}C$$

i.e.,

$$\mathbf{e}_c = -\mathbf{i}/\omega jC$$

This is illustrated in Fig. 43.

5. The Vector Analysis of Alternating Current Circuits.

It was shown in Para. 11, Part II (June, 1927), that Kirchhoff's law relating to the zero sum of currents meeting at a point in a network is equally true of the vectors used to represent any such set of alternating currents of the same frequency. Further, it will be found on reference to the proof there given that it applies also to the zero sum of any number of alternating potential differences of the same frequency. We may therefore say at once that Kirchhoff's first and second laws apply without any modification to the current and potential difference vectors of any single frequency alternating current network. The analysis of any such network is thus reduced to quite elementary vector algebra, in place of the systems of differential equations which arise from the ordinary scalar analysis of such problems.

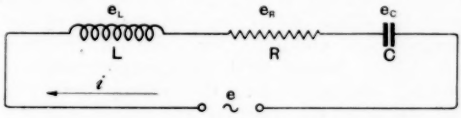


Fig. 44.

Consider, for example, the fundamentally important circuit shown in Fig. 44, *i.e.*, a pure resistance, a pure inductance, and a pure capacity in series with a source of alternating (sine wave) E.M.F., assumed to be of negligible internal resistance. Kirchhoff's second law gives at once the vector equation

$$\mathbf{e} + \mathbf{e}_R + \mathbf{e}_L + \mathbf{e}_C = 0$$

On the instant of closing the circuit certain transient phenomena will occur due to the fact that no system of finite mass (and inductance is the electrical equivalent of mechanical mass) can pass from one equilibrium condition to another instantaneously. This comparatively unimportant aspect of the matter will be considered later. When a steady state has been reached (generally in a fraction of a second) the current flowing in the circuit will be of the same character and frequency as the E.M.F., since no other than a current of this character can give rise to back E.M.F.s which will exactly balance the driving E.M.F. at every instant as required by the above equation. Therefore, assuming the E.M.F. to be

$$\mathbf{e}\cdot\boldsymbol{\nu} = \hat{e}\ cos\ \omega t$$

the current will be of the same frequency $\omega/2\pi$ and can thus be represented by the vector $\mathbf{i}$ of constant magnitude $\hat{\imath}$ and constant angular velocity ω. The above determined expressions for the back E.M.F.s in terms of the current can therefore be substituted in the potential difference equation, giving

$$\mathbf{e} - R\mathbf{i} - j\omega L\mathbf{i} - \frac{\mathbf{i}}{j\omega C} = 0$$

i.e.,

$$\left(R + j\omega L + \frac{1}{j\omega C}\right)\mathbf{i} = \mathbf{e}$$

or

$$\left\{R + j\left(\omega L - \frac{1}{\omega C}\right)\right\}\mathbf{i} = (R + jX)\mathbf{i} = \mathbf{e}$$

where X has been written for $\omega L - 1/\omega C$ for compactness. Thus, finally,

$$\mathbf{i} = \frac{\mathbf{e}}{(R + jX)}$$

This is the complete solution for the "steady state" alternating current in the circuit, and for most analytical purposes is the best form of representing it. It is probably the most important single result in the whole of alternating current theory—at least, as far as wireless telegraphy is concerned—and will therefore be examined in some detail.

In the first place, the scalar form of the solution can be written down at once. It has already been given in Para. 21, Part II (July, 1927). Expressing the operator $(R + jX)$ in the form $Z\epsilon^{j\phi}$, *i.e.*,

$$Z^2 = R^2 + X^2 \quad \text{and} \quad \phi = tan^{-1}\ X/R$$
$$\mathbf{i} = \mathbf{e}/(Z\epsilon^{j\phi}) = \epsilon^{-j\phi}\mathbf{e}/Z$$

The effect of this operator is to divide the magnitude of $\mathbf{e}$ by Z and to rotate it through an angle $-\phi$ in a positive direction, *i.e.*, through an angle ϕ in a negative (clockwise) direction. This is illustrated in Fig. 45. Therefore, since i is given by $\mathbf{i}\cdot\boldsymbol{\nu}$, and since $\mathbf{e}$ is such that $\mathbf{e}\cdot\boldsymbol{\nu} = \hat{e}\ cos\ \omega t$, we have

$$i = \mathbf{i}\cdot\boldsymbol{\nu} = \frac{1}{Z}\epsilon^{-j\phi}\mathbf{e}\cdot\boldsymbol{\nu} = \frac{\hat{e}}{Z}\ cos\ (\omega t - \phi)$$

Alternatively, since

$$\frac{1}{(R + jX)} = \frac{R}{Z^2} - \frac{jX}{Z^2}$$

$$\mathbf{i} = \frac{R}{Z^2}\mathbf{e} - \frac{jX}{Z^2}\mathbf{e}$$

which expresses **i** in terms of its components parallel and perpendicular to **e** (the vectors *OC* and *CA* in Fig. 45). The corresponding scalar form is, as shown in the Para. 21 referred to above

$$i = \mathbf{i}\cdot\nu = (R/Z^2)\mathbf{e}\cdot\nu - (X/Z^2)j\mathbf{e}\cdot\nu$$
$$= (R/Z^2)\hat{e}\cos\omega t + (X/Z^2)\hat{e}\sin\omega t$$

The back E.M.F.s corresponding to the current i are as shown by the vectors

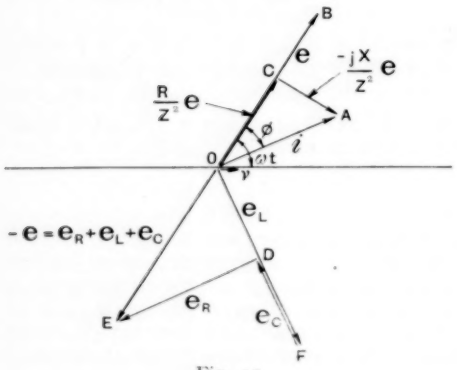


Fig. 45.

OF, *FD*, *DE* in Fig. 45. One of the chief advantages of the vector method of analysis is the facility with which the analytical expressions can be translated into pictures, in which form they are much easier to understand.

6. Generalisation and Nomenclature.

The group $(R+jX)$ is known as the "impedance operator" of the circuit. The magnitude of this operator, *i.e.*, Z or $(R^2+X^2)^{\frac{1}{2}}$ is called the "impedance." The angle $\theta = tan^{-1}\ X/R$ is called the "phase angle" of the impedance since it determines the angle between the current and E.M.F. vectors. The R term is called the resistance component, and the X term the reactance component of the impedance.

In general, the relationship between the current and E.M.F. vectors in any single frequency circuit, however complicated, will be an operator of the form $(R+jX)$. This follows from the fact that any combination of such operators can be represented as some other single operator of the same type. (See Para. 21 referred to above.) The terms defined above are, therefore, given a full generalised interpretation. Thus the R term, however constituted (and it will in most cases contain other than pure resistance terms) will be called the resistance component, and so on.

Some American writers have adopted a still further generalisation of the above nomenclature. The relationship between the current in any given branch of a network and the driving E.M.F. acting in the same or any other branch of the system is called the "transfer impedance" for the specified conditions. The full significance of the above terms will appear more clearly later on, when some rather more complicated systems have been discussed.

7. Impedance Variation.

Returning to the simple series circuit of Fig. 44, we are in practice much concerned with the effect of varying one or other of the component elements of the impedance $(R+jX)$. Notice first that impedance is a circuit characteristic, depending only on the electrical constants of the circuit and the frequency. A circuit is therefore most suitably discussed in terms of its impedance, without reference to any specific E.M.F.

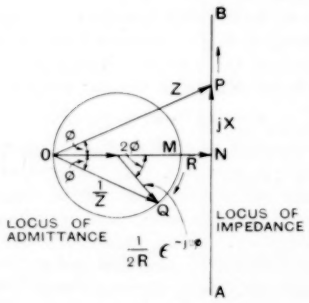


Fig. 46.

The impedance operator can be represented graphically as a vector by assuming the unit vector of reference ν as operand, though the operand need not appear in the analysis or diagrams. The general practice of representing the resistance and reactance components of an impedance by lines parallel to the x and y axes respectively is obviously equivalent to this assumption. On this

basis the impedance $(R+jX)$ can be represented by the line OP in Fig. 46. If now the term X is varied by varying the capacity R, L, and ω remaining constant, the point P will move along the line AB, which can thus be described as the locus of the impedance under the given conditions of variation. The minimum value of Z will clearly correspond to the point N for which X is O. Under these conditions **i** will reach its maximum value $\mathbf{e}/R$, the corresponding vector diagram being as shown in Fig. 47.

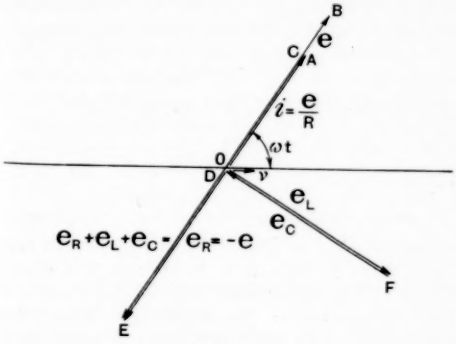


Fig. 47.

This condition is described as the "resonance" of the current in the circuit. In general, a minimum value of the impedance $Z=\sqrt{R^2+X^2}$ will be taken as the criterion of a resonance condition. Notice that in this particular case the resonance is associated with the vanishing of the reactance term, and the current at resonance is in phase with the E.M.F. (*i.e.*, the **i** and **e** vectors are parallel.) This is not always true of the resonance condition, and is therefore not the proper criterion of resonance.

Since Z is a minimum at resonance the condition can be stated in scalar form

$$dZ/dC = 0$$

with the proviso that the second differential coefficient shall be positive. (See Para. 11, Part III, Sept., 1927.) The reader can easily confirm that this will lead to the same conclusion in the above case—namely,

$$X = \omega L - 1/\omega C = 0$$

or

$$C = 1/\omega^2 L$$

The above describes resonance with respect to condenser variation. If frequency be taken as the variable the condition becomes

$$dZ/d\omega = 0$$

In the present instance the locus of Z is the same for condenser or frequency variation, and the resonance condition is therefore the same, too, *i.e.*,

$$\omega L - 1/\omega C = 0$$

but this again is not always true for the resonance condition, and the particular resonance considered must always be clearly specified.

8. The Current Locus. Circle Diagrams.

The straight line locus for the impedance found in the above simple case is typical of a large number of impedance variations due to variation of one of the magnitudes of the circuit. (In general, variation of the frequency gives rise to a more complicated locus. The loci just happen to be the same in the above case because it is a very simple form of circuit.) It will therefore be of interest to examine the variation of **i** in such a case. (This means, of course, the variation of the magnitude and phase of **i** relative to **e**, due to variation of the circuit magnitudes. The variation of i with respect to time is quite another matter, and must be clearly distinguished in the mind.) For the simple series circuit

$$z = R + jX = R(1 + jn)$$

where

$$n = X/R = \tan \phi$$

It will be assumed that R is constant and n variable. The current is given by

$$\mathbf{i} = \frac{\mathbf{e}}{z} = \frac{1}{R}\,\frac{1}{(1+jn)}\,\mathbf{e}$$

It has already been shown that the locus of z is the line AB in Fig. 46. It will now be shown that the locus of $1/z$ is the circle, shown in Fig. 46, *i.e.*, a circle of diameter $1/R$ centre at a distance $1/2R$ from O in the direction ν. By elementary algebra,

$$1/(1+jn) = \tfrac{1}{2}\left\{1 + \frac{1-jn}{1+jn}\right\}$$

and since

$$1 + jn = (1+n^2)^{\frac{1}{2}}\epsilon^{j\phi}$$

and

$$1 - jn = (1+n^2)^{\frac{1}{2}}\epsilon^{-j\phi}$$

$$1/(1+jn)=\tfrac{1}{2}\left(1+\frac{\sqrt{1+n^2}\,\epsilon^{-j\psi}}{\sqrt{1+n^2}\,\epsilon^{j\psi}}\right)=\tfrac{1}{2}(1+\epsilon^{-2j\psi})$$

Therefore

$$\frac{1}{z}=\frac{1}{2R}(1+\epsilon^{-2j\phi})=\frac{1}{2R}+\frac{1}{2R}\epsilon^{-2j\psi}$$

and since the line $(1/2R)\epsilon^{-2j\psi}$ as ϕ varies, moves round a circle of radius $1/2R$ as shown in Fig. 46, the proposition is proved. As the end of z moves along AB from N in the direction NB, the end of $1/z$, the reciprocal of the impedance operator, which is sometimes called the "admittance" operator, moves round the circle of diameter $1/R$ from M in the direction MQO. The maximum value of $1/Z$ is obviously $OM=1/R$.

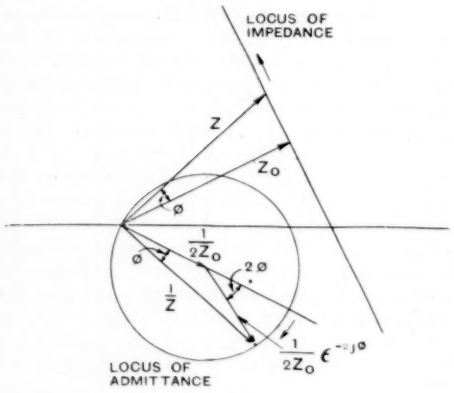


Fig. 48.

The above example is typical of a great number of circuit arrangements met with in wireless practice. In general, any impedance whatever which varies on a straight line locus with variation of one of the circuit magnitudes can be expressed in the form

$$z=z_0(1+jn)$$

where z_0 is not necessarily a resistance term, but may be of the form R_0+jX_0 where R_0 and X_0 are constants. The locus of z is a straight line perpendicular to and at the end of the line representing z_0. The locus of $1/z$ will be the circle $(1/2z)_0\,(1+\epsilon^{-2j\psi})$ where $n=\tan\phi$. (See Fig. 48.) This is the fundamental basis of the numerous circle diagrams which are met with in alternating current theory, and of which several examples have been described in this journal (*e.g.*, "Simple Resonance Curves and their Modification by Valve Circuits." Prof. E. Mallet, D.Sc., M.I.E.E., February, 1927. Also the series of articles by Dr. Dye on low frequency transformers.)

In all such cases the resonance condition is defined by $n=0$, and the minimum or resonance value of z is z_0.

A representative case of some practical importance is the resonance of the potential difference across the condenser for the series circuit described above. Since

$$\mathbf{e}_c=-\mathbf{i}/j\omega C$$

$$\mathbf{e}_c=-\mathbf{e}/\{j\omega C(R+j\omega L+1/j\omega C)\}$$

i.e.,

$$\mathbf{e}_c=-\mathbf{e}/Z$$

where

$$z=1+j\omega C\,(R+j\omega L)$$

(z can hardly be called an impedance in this case, but it amounts to much the same thing.) The expression of z in the standard form given above for a straight line locus requires a little ingenuity, but is quite easy to follow. Multiplying inside the bracket by $R-j\omega L$ and dividing outside by the same thing,

$$Z=\frac{1}{R-j\omega L}\{R-j\omega L+j\omega C(R^2+\omega^2L^2)\}$$

$$=\frac{R}{R-j\omega L}\left\{1+j\frac{\omega C(R^2+\omega^2L^2)-\omega L}{R}\right\}$$

which is in the standard form $Z_0(1+jn)$ where

$$z_0=R/(R-j\omega L)$$

and is constant with respect to variation of the tuning condenser C, and

$$n=\frac{\omega C(R^2+\omega^2L^2)-\omega L}{R}$$

and is a variable number on account of the variation of C. The locus of z is thus a straight line perpendicular to and at the end of the line representing $R/(R-j\omega L)$. The minimum value of z is therefore

$$z_0=\frac{R}{(R-j\omega L)}=\frac{R(R+j\omega L)}{R^2+\omega^2L^2}$$

the magnitude of which is

$$Z_0 = R/(R^2 + \omega^2 L^2)^{\frac{1}{2}} = R/\omega L$$

if R^2 is negligible compared with $\omega^2 L^2$, and the resonance value of C is given by $n=0$, *i.e.*,

$$C = L/(R^2 + \omega^2 L^2)$$

Notice that the maximum value of the P.D. across the condenser does not correspond exactly with the maximum value of the current in the circuit, an interesting and not generally realised fact. Actually, the difference will only be appreciable if R^2 is appreciable compared with $\omega^2 L^2$, which will not generally be the case. However, it is always a possibility. If R is negligible compared with L, the ratio of e_c to e (magnitude), *i.e.*, the ratio of the P.D. across the condenser to the E.M.F. in the circuit, is $\omega L/R$. This quantity is therefore a measure of the magnification of the E.M.F. due to resonance and in wireless applications it is obviously desirable to make this as large as possible. The difference between a good and a bad coil may amount to a stage of amplification. If R is small, as it should be, $\omega L/R$ varies very rapidly with R. This is the reason for the enormous increase in the sensitivity of reception obtainable by means of reaction, which reduces the effective value of R.

APPENDIX I.

Let $\mathbf{r}\cdot\nu = r \cos\theta$, r and θ being functions of t. Then

$$\begin{aligned}\frac{d(\mathbf{r}\cdot\nu)}{dt} &= \cos\theta\,\frac{dr}{dt} - r\sin\theta\,\frac{d\theta}{dt}\\ &= \cos\theta\,\frac{dr}{dt} + r\cos(\theta+\pi/2)\,\frac{d\vartheta}{dt}\\ &= \frac{(\mathbf{r}\cdot\nu)}{r}\,\frac{dr}{dt} + (j\mathbf{r}\cdot\nu)\,\frac{d\theta}{dt}\\ &= \left(\frac{1}{r}\,\frac{dr}{dt} + j\,\frac{d\theta}{dt}\right)\mathbf{r}\cdot\nu\\ &= \frac{d\mathbf{r}}{dt}\cdot\nu\end{aligned}$$

Examples.

1. Referring to the series circuit of Fig. 44,

$L = 100\mu\text{H}$ (*i.e.*, 10^{-4} henries).

$R = 10$ ohms.

$e = 10^{-2}\cos 2\pi \times 830 \times 10^3\, t$

Calculate :—

(*a*) The impedance operator corresponding to $C = 1{,}000\ \mu\mu\text{F}$.

(*b*) The instantaneous magnitude of the current in the circuit for this value of the tuning condenser.

(*c*) The magnitude of C which will give the maximum P.D. across the condenser.

(*d*) The ratio of the condenser P.D. to the E.M.F. at resonance.

2. Given that $i = \mathbf{i}\cdot\nu = \hat{\imath}\cos(\omega t - \phi)$ and $e = \mathbf{e}\cdot\nu = \hat{e}\cos\omega t$, show that the mean value of ie from $t=0$ to $t = 2\pi/\omega$ is $(\mathbf{i}\cdot\mathbf{e})/2$.

3. Show that for a damped oscillatory current of the form $i = \mathbf{i}\cdot\nu = \hat{\imath}\varepsilon^{-kt}\cos\omega t$, the back E.M.F.s due to a resistance R, an inductance L, and a capacity C are given by

$$\begin{aligned}\mathbf{e}_R &= -R\mathbf{i}\\ \mathbf{e}_L &= -(-k+\omega j)L\mathbf{i}\\ \mathbf{e}_C &= -\mathbf{i}/(-k+\omega j)C\end{aligned}$$

4. Show that the condition $\mathbf{e}_R + \mathbf{e}_L + \mathbf{e}_C = 0$ can be satisfied for the series circuit of Fig. 44 if the current is of the form $i = \hat{\imath}\varepsilon^{-kt}\cos\omega t$. Find the values of k and ω in terms of R, L and C.

Answers to Examples in October Issue.

1. $(k+\omega j)\mathbf{v}$; $\{(k^2-\omega^2)+2\omega jk\}\mathbf{v}$;
$\sqrt{k^2+\omega^2}\, v_0\varepsilon^{kt}\cos(\omega t + \psi + \tan^{-1}\omega/k)$;
$(k^2+\omega^2)v_0\varepsilon^{kt}\cos(\omega t + \psi + 2\tan^{-1}\omega/k)$

2. i. $(1/a)\log_e(ax+b) + const.$;
ii. $(ax/c) + \{(bc - ad/c^2\}\log_e(cx+d) + const.$;
iii. $\sec x + const.$;
iv. $-(1/a)\cot^{-1}(x/a) + const.$;
v. $\tan^{-1}\varepsilon^x + const.$

3. i. $(x^3/9)(3\log_e x - 1) + const.$;
ii. $x(\log_e x)^2 - 2x\log_e x + 2x + const.$;
iii. $x\tan^{-1}x - \frac{1}{2}\log_e(1+x^2) + const.$

5. i. 2 ; ii. 0 ; iii. π

Abstracts and References.

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PROPAGATION OF WAVES.

VARIATIONS IN HIGH FREQUENCY GROUND WAVE RANGES.—A. H. Taylor. (*Proc. Inst. Radio Engineers*, 15, 8, pp. 707-708.)

The results of experiments made by two naval ships on high frequency ground wave ranges indicate unexpected differences between day and night values, not predicted by present theory. The observations reported were taken on frequencies in the 12,000 and 16,000 kilocycle bands at distances between zero and 53 miles. The power of the transmitter was approximately 100 watts in the radio-frequency circuits, of which probably 60 per cent. went into the antenna. The daylight ground range in the 12,000 kilocycle band was found to be about 12 miles while the night range for the same frequency band was something over 50 miles. In the 16,000 kilocycle band the daylight ground wave range was not more than 3 miles, while the night range extended to 22 miles.

The author states that these results can only be interpreted by supposing some agency at work which produces a markedly greater absorption of the ground wave in the day-time than it does during the night hours; otherwise we should be forced to assume that we had to deal with some new kind of sky wave produced by a very low refracting or reflecting layer, which however is less likely owing to the absence of fading. Although these results are only in line with what we know of longer wavelengths, the differences between day and night ranges are then attributed to the different character of the sky waves in the two cases, while here we are dealing only with ground waves, the observations being taken well within the normal skip distance area.

A SUGGESTION OF A CONNECTION BETWEEN RADIO FADING AND SMALL FLUCTUATIONS IN THE EARTH'S MAGNETIC FIELD.—G. Breit. (*Proc. Inst. Radio Engineers*, 15, 8, pp. 709-723.)

The fading of radio signals has been repeatedly referred to as an interference effect between ground and sky waves. Recently, however, it has become obvious that more than simple interference is involved. Thus short-wave transmission has shown that beyond a certain range signals are received entirely by an overhead route, and yet, in spite of this fact, the signals show fading. While this may be explained, at least partly, as a general shift in the height of the reflecting layer, experiments on the reflection of waves from the upper atmosphere have proved that fading exists for the reflected waves quite independently of interference with the ground wave. One suggestion as to the possible cause of the fading has supposed the downcoming ray to be a result of the interference of two or more rays, and some simple calculations are given here in order to show how the interference conditions between these two rays may change. The state of polarisation of a wave returned by the reflecting layer is studied as a function of (*a*) small changes in the intensity of the earth's magnetic field, (*b*) small changes in the frequency of the wave, and (*c*) small changes in the ionisation of the atmosphere.

It is shown that, for the special electron distribution considered, appreciable effects on the intensity of the signal are to be expected for fluctuations in the earth's field of the order of 5 gammas, for changes in the frequency of the order of 1,000 cycles, and for changes in the ionisation of the order of one part in 5,000. (The number applies to $\lambda = 70$ metres.) The dependence of fading on range indicates that there is a certain range of maximum fading which is of the general order of 100 miles

PROPAGATION OF SHORT WAVES.—(*Nature*, 24th September, 1927, p. 454.)

Using an antenna only 25 ft. long and a wavelength of 32.8 metres, an experimental radio station in America has been heard all over the world.

The General Electric Co. has obtained interesting results with waves of 5 metres: these waves were found to show a shadow effect very similar to that produced by light; also they were picked up 32 miles away when a power of only 60 watts was employed. The experiments are being continued.

ÜBER DIE TEMPERATUR IN DEN HÖHEREN SCHICHTEN DER ATMOSPHÄRE (On the temperature of the upper layers of the atmosphere).—H. Petersen. (*Physik. Zeitschr.*, 28, 14, pp. 510-513.)

Assuming that magnetic storms are due to corpuscular radiation from the sun (β rays) which are caught or deviated by the earth's magnetic lines of force, and attributing to the radiation properties similar to those ordinary rapid β rays possess, and assuming further that this radiation is wholly or partly absorbed at great altitudes, the author shows that such a quantity of heat is developed as to considerably raise the temperature of the absorbing masses of air, and that the conditions are such as to account for the rise of temperature with increasing height that Gutenberg calculated from observations on explosion waves (0°C. at 40 kilometres, 20° at 50 kilometres, and 40° at 60 kilometres).

PENETRATION OF RADIO WAVES.—A. Eve, D. Keys and E. Denny. (*Nature*, 17th September, 1927, p. 406.)

In a letter to *Nature*, of 2nd July (these abstracts, *E.W. & W.E.*, September, 1927, p. 572), the desirability was expressed of obtaining information as to the extent to which radio waves can penetrate

the earth, and on 17th August, at the Caribou Mine, Colorado, the opportunity was offered of making satisfactory tests on this point. It was found that at a depth of 220 ft. below the surface, in a cross cut clear of wire, rails and pipes, KFEL Denver (248 metres) could be heard well from a loud-speaker, and then on proceeding to a depth of 550 ft., while carrier waves could be detected, no clear reception was possible in the morning, although in the evening KOA Denver (326 metres) was heard perfectly distinctly. In both cases the reception was by loop and maximum intensity was obtained when the loop pointed within a few degrees of Denver, about 50 miles away.

Previous experiments at the Montreal tunnel had shown that 40 metre waves were weak in penetrating power, that broadcasting waves were more effective, while longer waves of 10,000 metres surpassed both.

RESULTS OF EARTH-RESISTIVITY SURVEYS NEAR WATHEROO, WESTERN AUSTRALIA, AND AT EBRO, SPAIN.—W. Rooney and O. Gish. (*Terres. Mag. and Atmos. Elect.*, 32, 2, pp. 49-63.)

Description of an experimental investigation which shows that Watheroo, despite the presence of a surface layer of high resistivity sand, is a region of unusually high conductivity, comparable to a fresh-water area. The average value of the resistivity to depths of 100 metres is about 700 ohms per c.c., and to 600 metres a little over 5,000 ohms per c.c. The resistivity at Ebro, while lower at the surface, is considerably higher than that at Watheroo to depths of 300 metres or more, the average value being somewhat over 10,000 ohms per c.c. To depths of 100 to 300 metres, the current-density, as determined by combining resistivity results with records of potential gradient, differs very little at the two places.

ATMOSPHERICS AND ATMOSPHERIC ELECTRICITY.

ATMOSPHERICS AND THE ATMOSPHERE.—R. A. Watson Watt. (*Quart. Journ. Royal Meteorological Society*, 53, 222, pp. 169-172.)

Broadcast Talk No. 3, on "The Weather and its Ways," describing simply how atmospherics give information on the approach and direction of travel of a polar front and the weather consequently to be expected.

ON CLICKS AND GRINDERS OF ATMOSPHERICS.—H. Nagaoka. (*Proc. Imp. Acad., Japan*, 3, 2, 1927, pp. 64-67.)

The writer states that the clicks and grinders observed by radio-telegraphists appear to be similar electric disturbances in the upper atmosphere, but that while the former occur in the non-ionised region, the latter pass through the ionised layer. Accordingly clicks come in without much change of type, but grinders are greatly modified during the passage through the conducting medium and are accompanied by tails, tending to prolong the disturbance. A brief mathematical treatment based on Maxwell's equations for a slightly conducting medium is given elucidating this difference. The writer also states the possibility that grinders are caused by thunderstorms, the promiscuous waves generated by electric discharge passing through damp atmosphere, which is partially conducting, giving rise to the diffusive propagation found.

THUNDERSTORMS.—G. C. Simpson. (*Quart. Journ. Royal Meteorological Society*, 53, 222, pp. 172-176.)

The fourth broadcast talk on weather topics, explaining in simple language the origin of the electricity in a thunderstorm as due to the breaking up of raindrops, when they become charged with positive electricity, the surrounding air receiving the corresponding negative charge, also describing the mechanism of a thunderstorm and the part played by ascending currents of air.

REMARKABLE ELECTRICAL CONDITIONS ACCOMPANYING WEST TEXAS SAND STORMS.—E. George, W. Young and H. Hill. (*Physical Review*, September, 1927, p. 362.)

During West Texas sand storms the atmosphere is in a very unusual electrical condition. Severe shocks are sometimes received from radio antennæ, fence wires and automobiles. A radio antenna composed of stranded wire stretched at a height of about 73 ft. in an east and west direction between towers 282 ft. apart, formed the basis of preliminary measurements. Prevailing storms are from the west. Potentials of over 40,000 volts, as measured by spark gap between spherical electrodes, have been obtained and direct currents as high as 1.2×10^{-4} amperes measured.

PRELIMINARY NOTES ON ELECTROMOTIVE FORCES POSSIBLY PRODUCED BY THE EARTH'S ROTATING MAGNETIC FIELD AND AN OBSERVED DIURNAL-VARIATION OF THE ATMOSPHERIC POTENTIAL-GRADIENT.—G. Wait and H. Sverdrup. (*Terres. Mag. and Atmos. Elect.*, 32, 2, pp. 73-83.)

The current computed from the action of electromotive forces, due to the rotation of the earth's magnetic field, upon charged particles entering the upper atmosphere from the sun, shows a diurnal variation and annual variations of phase-angle and amplitude which are in remarkable agreement with corresponding variations of the atmospheric potential-gradient as actually determined from observations made at sea. Difficulty, however, is found in developing a physical basis to explain the relation between the two phenomena.

PROPERTIES OF CIRCUITS.

MODULATION IN VACUUM TUBES USED AS AMPLIFIERS.—E. Peterson and H. Evans. (*Bell System Technical Journal*, 6, 3, 1927, pp. 442-460.)

Recent developments in amplifier design tending toward more rigorous quality requirements have shown that the solutions of Van der Bijl and Carson are inadequate for certain purposes since they are based upon a convenient assumption which is not satisfied in fact. In particular, a detailed investigation of carrier current repeaters

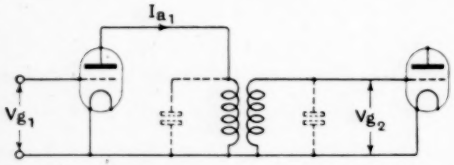
used for the simultaneous transmission of several channels, and upon which in consequence the modulation or cross-talk requirements are particularly severe, showed the modulation currents measured to be quite different from those specified by the theory, as was the law of variation of these currents with the circuit constants.

The cause of the discrepancy was found to reside in the neglect of the variation of the amplification factor (μ) with both plate and grid potentials. When the actual state of affairs was taken into account in the analysis by the application of a general method involving no assumptions, theory and experiment were found to be in good accord. The new expressions have been developed in terms of the amplification factor, the internal output resistance of the valve, and their differential parameters, which are involved in the representation of the characteristic valve equation by a double power series. Expressions for the current components are developed in terms of the coefficients of the series, and modifications of Miller's method for greater convenience and precision in determinations of valve characteristics are described from which the series coefficients may be evaluated.

Conclusions are drawn from the solutions as to desirable valve characteristics by which, for example, a single valve may take the place of two valves in push-pull connection. Finally, certain properties of different types of valve under conditions of maximum output power are compared on the basis of μ constant and μ variable.

DIE INDUKTIVE KOPPLUNG MIT PRIMÄRER UND SEKUNDÄRER ABSTIMMUNG IM ANSCHLUSS AN RADIORÖHREN. I. TEIL (Inductive coupling with primary and secondary tuning connecting valves).— H. Kafka. (*Zeitschr. f. Hochfrequenz.*, 30, 2, pp. 44-52.)

Investigation of the effect of inductive coupling with primary and secondary tuning for connecting two valves as shown below :—



The value and sharpness of tuning of the grid tension on the second valve produced by the coupling are of particular interest. It is shown that the inner resistance of the first valve has a very significant influence on the effect of the coupling. Inductive coupling with secondary tuning offers special advantages, since with loose coupling it enables the value and selectivity of the secondary grid current to be considerably increased. The influence of the degree of coupling on the value of the secondary grid current is represented by a locus diagram. For a certain degree of coupling the value of the secondary grid tension reaches an optimum value. Lastly, it is investigated to what extent the increase of selectivity obtainable with loose coupling can be utilised in practice. The experiments described are intended to be a starting point for the design of inductive couplings to connect valves in the case of high and intermediate frequency amplifiers.

AUDIO-FREQUENCY TRANSFORMERS.—J. M. Thomson. (*Proc. Inst. Radio Engineers*, 15, 8, pp. 679-686.)

A method for calculating the amplification curve of an audio-frequency transformer is developed in terms of the usual constants of the transformer and valve. The distributed capacity of the coils and the mutual capacity between the primary and the secondary coils are represented by lumped capacities. The exciting current of the transformer is neglected. As the equation for the amplification in the vector form is rather involved an approximate formula is developed and its limitations pointed out.

ZUR THEORIE DES WIDERSTANDSVERSTÄRKERS (On the theory of the resistance amplifier).—H. Dänzer. (*Zeitschr. f. Hochfrequenz.*, 30, 1, pp. 26-28.)

Mathematical investigation of the influence of the battery tension, the high ohmic resistance in the anode circuit, and the grid-leak resistance on the amplification ratio. The ratio is found to be a maximum in general when the resistance in the anode circuit is equal to twice the grid-leak resistance. The dependence of the maximum amplification ratio upon the factors coming into question is discussed.

RESISTANCE AMPLIFIERS.—P. Tyers. (*Electrical Review*, 9th September, 1927, p. 416.)

Some of the statements in this article are discussed by Mr. F. Phillips in the *Review* of 23rd September, p. 501.

LABILITEIT VAN EEN UIT *n* TRIODEN BESTAANDE VERSTERKER MET INACHTNAME VAN DE INTERELECTRODEN-CAPACITEITEN (Instability of an amplifier consisting of *n* triodes, taking account of the inter-electrode capacities.—K. Posthumus. (*Tijds. Nederland. Radiogenootschap*, 3, 3/4, 1927, pp. 106-112.)

Mathematical consideration of the influence of grid-anode capacity coupling on amplifier stability, in some simple cases, neglecting grid currents, parasitic back-couplings, and curvature of the characteristic.

FREQUENCY DEMULTIPLICATION.—B. van der Pol and J. van der Mark. (*Nature*, 10th September, 1927, pp. 363-364.)

Account of the utilisation of the remarkable synchronising properties of relaxation-oscillations to effect frequency demultiplication, a demultiplication of frequency up to the ratio 1 : 1/200 having been obtained.

EENIGE OPMERKINGEN OVER RELAXATIETRILLINGEN (Some remarks on relaxation oscillations).—H. O. Roosenstein. (*Tijds. Nederland. Radiogenootschap*, 3, 3/4, 1927, pp. 90-93.)

A mathematical note referring to Dr. van der Pol's paper (these Abstracts, *E.W. & W.E.*, August,

1927, p. 506), showing that relaxation oscillations can occur in a system where inductance is entirely absent, and that the inductance of the leads has no influence on the manner of oscillation of a multivibrator.

Über Kippschwingungen in gekoppelten Schwingungskreisen mit veränderlicher Selbstinduktion (On "tilting" oscillations in coupled oscillatory circuits with variable inductance).—R. Mayer and F. Sammer. (*Telefunken-Zeitung*, 8, 45/46, pp. 73-76.)

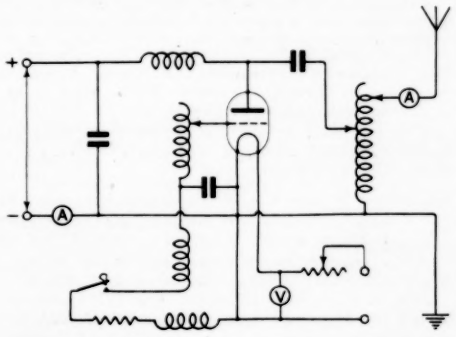
Account of the stationary current variations in coupled oscillatory circuits with an iron-containing inductance in the second circuit.

On the Design of an Inductance Coil for Audio-Frequencies having an Iron Core with an Air Gap.—H. Nukiyama and K. Nagai. (*Tohoku Univ. Technol. Reports*, 6, 3, 1927, pp. 1-10.)

TRANSMISSION.

Korte Golfzenders in het algemeen en die voor de verbinding Nederland-Curaçao in het bijzonder (Short wave transmitters in general and those for the service, the Netherlands-Curaçao in particular).—G. Schotel. (*Tijds. Nederland. Radiogenootschap*, 3, 5, April 1927, pp. 113-133.)

After explaining the need for a specially constructed transmitter for short waves, the author first discusses the circuit-arrangement in which the



antenna forms part of the anode circuit, as in the Huth-Kühn patent, and then goes on to consider the arrangement with indirect coupling. It is found, in the first case, that it is impossible, below a certain wavelength, to make the impedance of the valve circuit (including the capacity of the valve) sufficiently great for the triode to have good efficiency as a converter of direct into alternating current; and in the second case, that the loss due to the intermediate circuit detracts considerably from the total efficiency—by which is understood the ratio of the power radiated to that supplied to the anode. The author then describes his search for a system with good over-all efficiency: the circuit-arrangement developed is shown in the diagram.

Particulars with numerical values are given of the transmitters for communication between the Netherlands and Dutch West Indies, also a photograph of the Curaçao transmitter (wave range 20-80 metres).

Über Modulationslinien beim Röhrensender (On modulation lines in the case of valve transmitters).—W. S. Pforte. (*Zeitschr. f. Hochfrequenz.*, 30, 1, 1927, pp. 6-9.)

Modulation lines are curves representing the dependence of the antenna- or oscillatory circuit current upon the variation of individual circuit elements. These curves are plotted for the heating circuit, the grid circuit (grid tension, grid resistance, back coupling, back coupling capacity, back coupling parallel resistance), and the anode circuit (anode tension, anode resistance and oscillatory circuit resistance), mostly with two parameters: the back coupling and the tension applied to the grid.

Formulas for the Calculation of the Capacity of Antennas.—F. Grover. (*Proc. Inst. Radio Engineers*, 15, 8, pp. 733-736.)

A collection of formulæ covering different antenna types, together with tables of constants to aid in the calculations, and tables of the capacity itself for certain simple antenna systems, has been issued as a letter circular by the Bureau of Standards. This paper discusses the methods utilised and assumptions made in deriving the formulæ.

Calculations of the Polar Curves of Extended Aerial Systems.—E. Green. (*E.W. & W.E.*, October, 1927, pp. 587-594.)

Calcul des Constantes Electriques et Mécaniques des Antennes Pseudo-symétriques avec Application aux Antennes genre FL (Calculation of the electrical and mechanical constants of pseudo-symmetrical antennæ, with application to antennæ of the FL type).—M. Stern. (*L'Onde Electrique*, 6, 67, July, 1927, pp. 304-321.)

The general method of treating problems of pseudo-symmetrical antennæ is recalled. Two concrete cases are studied for antennæ of the FL type, one with six wires and the other with ten, assuming certain practical data known. The validity of the hypotheses is discussed and the determination of electrical and mechanical constants by calculation is considered in detail.

Zieherscheinungen beim Lichtbogensender (Oscillation hysteresis phenomena in the case of arc transmitters).—H. Winkler. (*Zeitschr. f. Hochfrequenz.*, 30, 1, 1927, pp. 1-5.)

The results are given of an investigation of the phenomena of oscillation hysteresis occurring in arc transmitters with close antenna coupling. The correctness of the explanation of the phenomena in accordance with Rogowski's theory of inductively coupled circuits was proved by quantitative experiments.

PROPAGATION OF PERIODIC CURRENTS OVER A SYSTEM OF PARALLEL WIRES.—J. Carson and R. Hoyt. (*Bell System Technical Journal*, 6, 3, 1927, pp. 495-545.)

Mathematical discussion, some results of which are applicable to wave antenna problems.

RECEPTION.

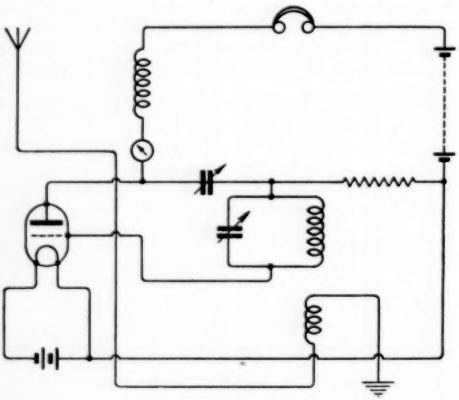
SUR LES APPLICATIONS D'UN TYPE PARTICULIER D'AMPLIFICATEUR À RÉACTION (On the applications of a particular type of amplifier with reaction).—P. Lejay. (*Comptes Rendus*, 185, pp. 500-502, 29th August, 1927.)

Account of the employment of very simple apparatus, comprising only a fixed and a variable resistance besides the valves, for detecting, eliminating atmospherics, amplifying, or as an oscillator.

THE AMPLIFICATION OF SMALL CURRENTS BY MEANS OF THE THERMO-RELAY AND THE PHOTO-ELECTRIC CELL.—J. Taylor. (*E.W. & W.E.*, October, 1927, pp. 627-633.)

NOUVEAU MONTAGE DE TRIODE DÉTECTRICE À REACTION (New circuit-arrangement for a detecting valve with reaction).—M. Caillat. (*L'Onde Electrique*, 6, 67, July, 1927, pp. 322-324.)

The circuit is here shown diagrammatically :—



For frame reception with very short waves, there is a further diagram showing the frame consisting of a single turn constituting the inductance of the oscillatory circuit.

VALVES AND THERMIONICS.

THE CHARACTERISTICS OF THERMIONIC RECTIFIERS.—C. L. Fortescue. (*Proc. Phys. Soc.*, 39, 4, 1927, pp. 313-317.)

In a paper published in 1919 (*Proc. Phys. Soc.*, 31, 5, 319) methods were given for calculating the behaviour of thermionic rectifiers working at high voltages and employing filaments with sharply defined saturation values for the emission current. The present paper extends the results there obtained to the case of rectifying valves working at low voltages with unsaturated electron currents, and briefly discusses the most economic conditions.

THE SHIELDED PLATE VALVE AS A HIGH-FREQUENCY AMPLIFIER.—R. T. Beatty. (*E.W. & W.E.*, October, 1927, pp. 619-625.)

THE NEW SCREENED VALVE.—N. W. McLachlan. (*Wireless World*, 31st August and 7th September, pp. 260 and 307, respectively.)

Discussion of the advantages and performance of the new screened valve with circuit calculations for tuned anode coupling.

SENDE-UND GLEICHRICHTER-RÖHREN MIT WASSERGEKÜHLTER ANODE (Transmitting and rectifying valves with water-cooled anode).—G. Jobst and S. Ganswindt. (*Telefunken-Zeitung*, 8, 45/46, pp. 64-73.)

Various data of water-cooled valves are given and compared with those of valves cooled by radiation. The article is well illustrated.

SILICA VALVES IN WIRELESS TELEGRAPHY.—H. Morris-Airey, G. Shearing and H. Hughes. (*Journ. Inst. Elect. Eng.*, August, 1927, pp. 786-790.)

A Paper read before the Wireless Section, 4th May, 1927, an abstract of which appeared in *E.W. & W.E.* for June.

COOLED-ANODE VALVES, AND LIVES OF TRANSMITTING VALVES.—W. S. Picken. (*Journ. Inst. Elect. Eng.*, August, 1927, pp. 791-812.)

A Paper read before the Wireless Section, 4th May, 1927, an abstract of which appeared in *E.W. & W.E.* for June.

THE HOLWECK DEMOUNTABLE TYPE VALVE.—C. F. Elwell. (*Journ. Inst. Elect. Eng.*, August, 1927, pp. 784-785.)

A Paper read before the Wireless Section, 4th May, 1927, an abstract of which appeared in *E.W. & W.E.* for June.

AMERICAN RADIO VALVE DEVELOPMENTS.—F. H. Engel. (*Electrical Review*, 9th September, 1927, p. 443.)

Abstract of Paper presented at the recent National Electrical Manufacturers' Convention, U.S.A.

Details are given of some new "Radiotrons," which are believed to meet the latest demands of receiving-set design.

SURFACE LAYERS ON TUNGSTEN PRODUCED BY ACTIVE NITROGEN.—C. Kenty and L. Turner. (*Nature*, 3rd September, 1927, p. 332.)

Preliminary account of experiments from which some of the conclusions drawn are as follows: A clean tungsten surface at a dull-red heat, if placed in an atmosphere of nitrogen, activated either by a condensed discharge or by an electron bombardment at more than 22 volts, becomes covered with a nitrogen layer of the order of one atom deep. The effect of this layer, at this comparatively low temperature is to cool the surface. At relatively high temperatures, the same layer is probably

so unstable that only a small fraction of the surface can be covered at any one time, but it acts to increase the work function.

Über den Formierungsprozess in Oxydkathodenröhren (On the process of activation in valves with oxide cathodes).—F. Detels. (*Zeitschr. f. Hochfrequenz.*, 30, 1 & 2, 1927, pp. 10-14 and 52-59.)

A Paper divided into the following eight sections:

1. Previous work on the problem.
2. The process of activation.
3. Proof of the decomposition of the oxide.
4. Determination of the temperature of the incandescent filament.
5. Determination of A and Φ.
6. Conclusions drawn.
7. Mathematical calculations of temperature distribution.
8. Summary.

The experiments show that the gas escaping during the process of activation is oxygen, liberated through electrolytic decomposition of the oxide on the filament, and that it is the metal set free that is the emitting substance, and not the oxide. After determining the temperature of the filament by two different methods: that of raising the resistance and that assuming Maxwell's distribution of electronic velocity, the variation of the work of emission Φ and emitting capacity A was found from the Davisson saturation current formula as a function of the duration of the activation.

Both qualities were shown to decrease, Φ slowly and A very rapidly. Increasing the saturation current therefore centres on decreasing the work of emission. Attempts to increase the emitting capacity by nickelling and coppering the filament wire had the effect surmised, but not to the extent expected.

The Rates of Evaporation and the Vapour Pressures of Tungsten, Molybdenum, Platinum, Nickel, Iron, Copper and Silver.—H. Jones, I. Langmuir and G. Mackay. (*Physical Review*, 30, 2, August, 1927, pp. 201-214.)

Over de Diffusie van Thorium door Wolfram (On the diffusion of thorium through wolfram).—P. Clausing. (*Physica*, 7, 6, 1927, pp. 193-198.)

Evidence is given for the conclusion that the diffusion of *Th* does not occur through the *W*-lattice, but along the boundaries of the *W*-crystals; and the *Th*-layer on the outside of the *W*-wire is believed to be formed by a surface-mobility of the *Th*-atoms.

DIRECTIONAL WIRELESS.

Rahmen- und Goniometerpeilanordnungen (Frame and goniometer arrangements for obtaining bearings).—A. Esau. (*Zeitschrift f. Hochfrequenz.*, 29, 6, pp. 181-190, and 30, 1, pp. 15-23, 1927.)

A paper in three sections calculating the directional errors for the most different antenna forms in relation to polarisation and angle of incidence and showing how to obtain an arrangement that is free from directional error under all circumstances.

The first section deals with the three rotatable antenna arrangements: the frame, double antenna, and V antenna; the results being summarised as follows: The frame gives bearings without error only when the waves are incident horizontally, or with inclined waves when the polarisation is normal. If the plane of polarisation has become rotated, errors come into the observation whose values depend both on the inclination and polarisation of the wave. The double antenna arrangement yields correct bearings under all circumstances, provided that the ratio $\frac{\text{distance apart}}{\text{wavelength}} < 1$. The same is true for the rotatable V-antenna. Up to now only the first arrangement has found practical application.

The second section considers the goniometer consisting of two crossed frames. It is found that error is absent only when either the polarisation is normal or the incidence horizontal. Waves that are inclined and at the same time not polarised normally give rise to errors the magnitude of which increases with increasing angle of inclination, reaching a maximum for the angle of 90 degrees. The error attains its greatest value irrespective of the angle of inclination when the horizontal angle of incidence falls within one of the two frame planes; it disappears when this angle equals 45 degrees. With the angle of inclination kept constant, the error increases with the size of the angle by which the plane of polarisation deviates from the normal. If the two frame antennæ do not cross exactly at an angle of 90 degrees, an error is introduced even with horizontal wave incidence, the value of which increases with the divergence. Much the same is true when the two field coils of the goniometer are not exactly perpendicular to one another. While in the former case the instrument only reads correctly for an angle of incidence 90 degrees, in the latter case the same is true for an angle of 0 degree; also the error in the two cases is in opposite directions. If the two causes act together, the resultant error is somewhat less than that due to either separately, also the direction of the error changes sign between the angles of incidence 0 and 90 degrees. If the frames are not exactly equal, an error is introduced, even with horizontal wave incidence, whose value increases with the degree of their inequality. Error is a maximum with an angle of incidence of 45 degrees.

The third section shows that the only arrangement free from directional error, whatever the polarisation and angle of incidence, is one consisting of two pairs of non-directional antennæ at right angles to one another, provided the ratio d/λ is made as small as possible, any way, less than $\frac{1}{10}$ (d is the distance apart of the antennæ). If this condition is not fulfilled then errors occur even with horizontal wave incidence as is not the case with frames. The errors are greatest for the angles of incidence 22.5 or 67.5 and decrease in size as the angle of inclination becomes larger and the ratio d/λ smaller. When the pairs of antennæ and the field coils are not exactly perpendicular to one

another, error is introduced in just the same way as with frame goniometers.

The results calculated here show the same differences for the antenna forms as were found by Buchwald and Baldus in their observations of bearings finding on aircraft. (*Jahrb. d. drahtl. Telegraphie u. Telephonie*, 15, 1920, p. 214.)

Über das Peilen von Drehfeldern mit Rahmen und Hilfsantenna (Taking bearings on rotating fields with frame and auxiliary antenna).—F. A. Fischer. (*Zeitschr. f. Hochfrequenz.*, 30, 1, pp. 23-25.)

According to recent views on the propagation of wireless waves, directional errors are not caused through the ray travelling along the earth's surface being deviated from its original direction but through one or more space rays arriving simultaneously at the receiver, with any phase, direction and polarisation (Smith-Rose and Barfield, *J.I.E.E.*, 64, 1926, 831), and which are more or less reflected according to the wavelength and the electric constants of the ground (conductivity, dielectric constant and permeability).

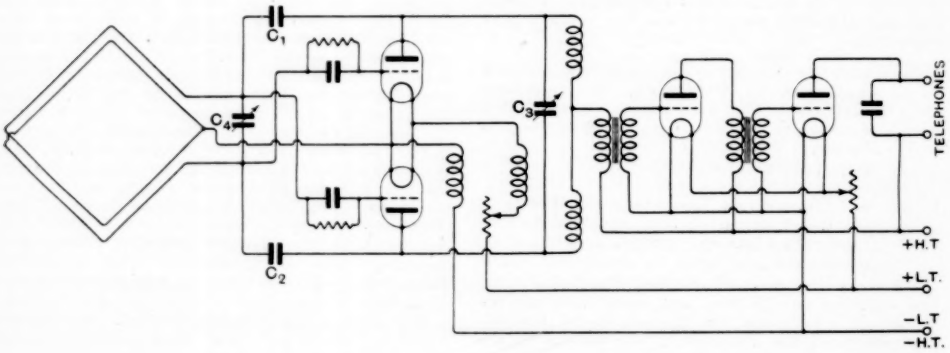
Now Heilitag has thoroughly investigated the directivity of an ideal frame antenna influenced simultaneously by two radio waves of the same frequency, but differing in direction, intensity and phase (*Jahrb. d. drahtl. T. u. T.*, 21, 1923, p. 77) and he arrived at the result that no conclusion can be drawn as to the magnitude of the directional error and the reliability of the observations from the quality of the minimum, as is shown again here. In general the direction of the long axis of the ellipse is indicated with a blurred minimum. Heilitag assumed an ideal frame, but in practice it is impossible to construct a frame direction finder so symmetrically that there is no small "antenna effect." This introduces a disturbing E.M.F., in addition to the frame E.M.F., and 90 degrees out of phase with it. To compensate for this, Telefunken employs the "auxiliary antenna," a small vertical wire antenna, which must be sufficiently mistuned to the arriving wave, since it has to induce an E.M.F. 90 degrees out of phase with it in the frame circuit. The auxiliary circuit is so coupled to the frame circuit that it produces in it an E.M.F. which just wipes out the E.M.F. of the antenna effect, thereby making the blurred minimum an absolute one.

It is investigated mathematically here how the combination of frame and auxiliary antenna takes bearings on a rotating field. The calculation is first made assuming the earth to be a perfect reflector, and then for the general case. It is found that the frame direction finder with auxiliary antenna, when taking bearings on elliptical rotating fields produced through several rays meeting at the receiver, always gives an absolute minimum, whose direction does not coincide with that of the long axis of the rotating field.

While the ideal frame always gives the direction of the long axis of the ellipse, even when the rotating fields are due to the presence of a back radiated field (as from the masts of a ship), with the frame direction finder and auxiliary antenna in this case, it depends on the coupling between the auxiliary antenna and the back radiation field. In practice, the auxiliary antenna is usually so placed that this coupling is absent, and then the antenna compensates the component of the back radiation field that is 90 degrees out of phase with the incident wave.

Über Rahmenpeilen mit kurzen Wellen (Taking bearings with a frame on short waves).—F. Michelssen. (*Telefunken-Zeitung*, 8, 44, pp. 66-71.)

A frame receiver for position-finding is described for the wave-range 18-120 metres, and the difficulties in developing it are pointed out. A circuit diagram is shown below.



Experimental investigation over land showed that sharp minima could be obtained only for short distances. Over sea, reliable bearings by day and night could not be guaranteed for more than 10 sea miles from the transmitter—and that with wooden ships. For iron ships, the errors with short waves are so exceptionally large and depend so markedly upon the frequency that their compensation is too difficult. It is concluded that the use of short waves for direction-finding is not practicable

and that the proved range of 600-1,100 metres must be retained at least for the present.

ÜBER DIE PEILBARKEIT KURZER WELLEN AUF SEE BEI TAG UND NACHT (On the possibility of taking bearings on short waves at sea, by day and night).—F. Michelssen. (*Telefunken-Zeitung*, 8, 45/46, pp. 96-99.)

Further details are given of the experiments described in the previous number of this periodical, which led to the conclusion that the employment of short waves for taking bearings is not practicable. It is stated that the best service a frame direction finder with auxiliary antenna can render at sea is in the investigation of propagation phenomena (for determining the distances to which the surface wave, with the small absorption over sea-water, affects the receiver simultaneously with a space wave, and then perhaps at what distance, and time of day, silent zones occur over sea for the various wavelengths.

DIE ERMITTELUNG WAHRER FUNKSTRAHLRICHTUNGEN AUF LANDPEILSTATIONEN (Finding the true direction of a radio ray with a land direction-finder).—D. Nitzsche. (*Telefunken-Zeitung*, 8, 44, pp. 72-78.)

When direction-finding on land, the incoming wave is frequently deviated by conducting structures in the neighbourhood of the receiver, as always happens on board ship. In order to determine what corrections have to be made for these systematic errors, the simplest way is to take observations on known stations, and compare the results obtained with the true directions. The true directions have therefore to be found. This paper discusses the construction and use of the various charts and finds that a gnomonic projection is best for direction-finding, as on it great circles are drawn as straight lines and a great circle is the shortest distance between two points on a sphere and consequently the path taken by radio waves. The fact that the projection is not angle true only comes in for distances that are at present too great for reliable direction-finding. In the absence of a suitable chart, the direction has to be calculated, which is explained with the help of an example. Thus with the correction to be applied to the reading observed in every direction known, the receiver is in a position to find the true direction of any unknown transmitting station.

WEITERER AUSBAU DER FUNKBESCHICKUNGSTHEORIE (Further development of the quadrantal error theory).—F. A. Fischer. (*Telefunken-Zeitung*, 8, 44, pp. 63-66.)

The general equation for quadrantal errors is given (*cf. T.-Z.* No. 42). It is found that while a ship's listing mostly reduces quadrantal error, its inclination lengthwise increases it. An important result of the investigation for frame compensation theory is that a frame compensates independently of the frequency when it has the same natural oscillation and the same damping as the ship (*cf. Annalen der Hydrographie*, 1926, xi, or *E.T.Z.*, 1926, 50).

BEITRAG ZUR KOMPENSIERUNG DES ANTENNENEFFEKTES VON PEILERN (Contribution to the compensation of the antenna effect in direction-finders).—F. Michelssen. (*Telefunken-Zeitung*, 8, 44, pp. 71-72.)

In order to obtain a sharp minimum, the E.M.F. produced by the electric vector due to unavoidable dissymmetry in the receiver relatively to earth must be compensated. Two methods for achieving this are explained here: coupling to the frame circuit (1) an aperiodic antenna, or (2) a tuned auxiliary frame.

EIN GEGENAZIMUTALER KARTENENTWURF ZUR ERMITTLUNG DER AZIMUTGLEICHEN FÜR KLEINE UND MITTLERE ENTFERNUNGEN VON DER FUNKBAKE (Construction of an azimuthal chart for determining lines of the same azimuth for small and medium distances from the transmitter).—W. Immler. (*Zeitschr. f. Hochfrequenz.*, 30, 2, pp. 60-66.)

ELECTROMAGNETISCHE STRALENBUNDELS (Electromagnetic beams).—R. Mesny. (*Tijds. Nederland. Radiogenootschap*, 3, 3/4, 1927, pp. 49-66.)

Discussion of the properties of curtain antennæ for producing beams. The advantages of employing a single wire bent in the shape of the Greek key-pattern are described, the polar curves obtained being compared with those produced by other methods. With regard to rotating beacons, a unidirectional system consisting of two groups of two antennæ is stated to be the simplest way of obtaining the sharpest beam (see *L'Onde Electrique* for last May, pp. 181-199, these Abstracts *E.W. & W.E.*, August, 1927, p. 508).

MEASUREMENTS AND STANDARDS.

DISTORTION OF RESONANCE CURVES OF ELECTRICALLY-DRIVEN TUNING FORKS. — E. Mallett. (*Proc. Phys. Soc.*, 39, 4, 1927, pp. 334-358.)

The dissymmetry may be of two different types: firstly, that of a device acting as a single system which with larger forces and consequently larger amplitudes gives a distorted resonance curve due to the non-linearity with amplitude of the forces brought into play, and secondly a "coupled circuit" effect occurring at small amplitudes as well as large ones. The conclusion is drawn that for any purpose where it is important that the frequency of the fork should be as absolutely constant as possible, it is necessary to work with very small amplitudes of vibration.

NOTES ON THE TESTING OF AUDIO-FREQUENCY AMPLIFIERS.—E. T. Dickey. (*Proc. Inst. Radio Engineers*, 15, 8, pp. 687-706.)

The more necessary of the various points which must be tested in examining the performance of audio-frequency amplifiers are outlined, and a method of test procedure, found by the author to have desirable characteristics from the points of view of accuracy, speech, and simplicity of operation, is described. The method permits the complete curve of amplification *v.* frequency for the

amplifier under test, to be drawn directly by the test equipment in a very short time. A type of valve voltmeter found convenient for measurement of amplifier output potential is described. Amplifier wave form distortion and overloading are discussed, and methods for testing them recommended.

EINE NEUE METHODE ZUR MESSUNG DES DÄMPFUNGSWIDERSTANDES VON SCHWINGUNGSKREISEN (A new method of measuring the damping resistance of oscillatory circuits).—L. Stürmer. (*Zeitschr. f. Hochfrequenz.*, 29, 6, pp. 192-194.)

Description of a method applicable to antenna and oscillatory circuit resistances greater than 3-8 ohms in the case of short waves (300-1,000 metres) and greater than 3-5 ohms with long waves (above 1,000 metres). The average accuracy is 1-5 per cent. according to the value of the resistance sought. The result is practically independent of the coupling with the auxiliary circuit, and the method requires very little energy for the measurement, and is adaptable to any valve wavemeter with self-excitation.

STANDARD FREQUENCY DISSEMINATION.—M. S. Strock. (*Proc. Inst. Radio Engineers*, 15, 8, pp. 727-731.)

A paper dealing with standard frequency dissemination through the medium of radio transmission, for which the Bureau of Standards has three avenues: standard frequency transmissions; selection by actual frequency measurements of certain transmitting stations which are termed "standard frequency stations"; and the selection of certain "constant frequency stations" which maintain their frequencies close to the licensed values.

LA MESURE EXACTE ET PRÉCISE DES LONGUERS D'ONDE DANS LES STATIONS D'EMISSION (Precise measurement of the wavelengths of transmitting stations).—R. Braillard and E. Divoire. (*L'Onde Electrique*, August, 1927, pp. 357-387.)

Detailed study of an accurate wavemeter, including the value of the inductance and capacity, the indicating circuit and the method of calibration.

HET METEN VAN CAPACITEITEN EN HET AANTOONEN VAN CAPACITEITSVERANDERINGEN (The measurement of capacities and indication of change of capacity).—J. W. Alexander. (*Physica*, 7, 6, 1927, pp. 213-221.)

The method of measurement is a substitution method: first, the total capacity is found of a variable precision condenser connected in series with a condenser of known capacity, the unknown capacity is then connected in parallel with the known condenser and the total capacity again measured, and from the alteration of the precision condenser required to give the previous total capacity the unknown capacity is determined. The capacity is made to form part of an oscillating circuit and the turning point of the resonance curve employed to indicate adjustment. Comparisons with Whiddington's method (*Phil. Mag.*, 40, 634, 1920), which has the same object in view, shows the method to be not more sensitive than his. The effect of error in the readings is calculated.

MEASUREMENT OF INDUCTANCE BY THE SHIELDED OWEN BRIDGE.—J. Ferguson. (*Bell System Technical Journal*, 6, 3, pp. 375-386.)

The investigation shows that the Owen Bridge is well adapted to the accurate measurement of inductance and effective resistance to above 3,000 cycles. The construction of a shielded bridge for audio frequencies is described and a theoretical discussion given.

NOTE ON PIEZO-ELECTRIC GENERATORS WITH SMALL BACK ACTION.—A. Hund. (*Proc. Inst. Radio Engineers*, 15, 8, pp. 725-726.)

Brief discussion of three circuit arrangements.

EINE PRAKTISCHE FASSUNG FÜR PIEZO-QUARTZPLATTEN (A practical mounting for piezo-quartz plates.—V. Gabel. (*Zeitschr. f. Hochfrequenz.*, 29, 6, pp. 194-195.)

Unfortunately the frequency of a quartz plate depends upon the width of the air-gap between the quartz surface and the electrodes and, as found by Dr. Dye, the smaller the air-gap, the more pronounced the effect; also the plate being situated asymmetrically between the electrodes exerts a certain influence. The author consequently made attempts to do away with the air-gap altogether by coating over the two surfaces of the quartz with a thin layer of metal. He describes the process of first chemically silvering the plate and then coppering it electrolytically. Plates thus treated can be very simply mounted in a case of any insulating material.

A WIRELESS WORKS LABORATORY.—P. K. Turner. (*Journ. Inst. Elect. Engineers*, September, 1927, pp. 881-902.)

Paper read before the Wireless Section, 18th May, 1927, pp. 881-902, an abstract of which appeared in the July number of *E.W. & W.E.*, pp. 422-429.

SUBSIDIARY APPARATUS AND MATERIALS.

H.T. FILTER CIRCUITS FOR D.C. MAINS.—J. Owen Harries. (*E.W. & W.E.*, October, 1927, pp. 613-618.)

A NEW FREQUENCY TRANSFORMER OR FREQUENCY CHANGER.—I. Koga. (*Proc. Inst. Radio Engineers*, 15, 8, pp. 669-678.)

Explanation of the employment of a triode oscillator to obtain alternating current having a frequency which is any fractional value of the frequency of a supplied current. The phenomenon seems to be due to the "attraction" of two nearly equal frequencies occurring in the triode circuit and to the non-linear characteristic of a triode.

RADIO-FREQUENCY TRANSFORMERS: THEIR APPLICATION TO SCREENED VALVES.—N. W. McLachlan. (*E.W. & W.E.*, October, 1927, pp. 597-600.)

THE PROPERTIES OF THE CIRCLE DIAGRAM FOR TELEPHONIC FREQUENCY INTERVALVE TRANSFORMERS.—F. E. Hackett. (*E.W. & W.E.*, October, 1927, pp. 601-604.)

DIE FREQUENZSTEIGERUNG VERMITTELS STARK GESÄTTIGTER TRANSFORMATOREN (Frequency raising by means of highly saturated transformers).—M. Osnos. (*Telefunken-Zeitung*, 8, 45/46, pp. 76-82.)

LOUD-SPEAKER DIAPHRAGMS.—N. W. McLachlan. (*Wireless World*, 21st September, 1927, pp. 357-361.)

The second article of a series, discussing the air pressure and energy distribution in the space surrounding the diaphragm.

THE KONE LOUD-SPEAKER.—S. Hill. (*Electrical Communication*, 6, 1, 1927, pp. 24-28.)

SOME MEASUREMENTS OF A "STALLOY" CORE WITH SIMULTANEOUS D.C. AND A.C. EXCITATION.—L. B. Turner. (*E.W. & W.E.*, October, 1927, pp. 594-596.)

STATIONS: DESIGN AND OPERATION.

THE SAINT-HUBERT AERODROME WIRELESS STATION.—(*Wireless World*, 28th September, 1927, pp. 447-448.)

Description of the station, recently opened by the Belgian Department of Aeronautics, which has been constructed to deal with the traffic of two important international air routes, viz., Amsterdam-Brussels-Basle and Paris-Cologne-Berlin.

BROADCASTING IN INDIA.—V. A. M. Bulow. (*Wireless World*, 7th September, 1927, pp. 311-314.)

A description of the Bombay station.

WAVELENGTH AND POWER OF EUROPEAN STATIONS.—(*Wireless World*, 28th September, 1927, pp. 149-454.)

A list, as complete as possible, of European broadcasting stations in order of wavelength, with a map giving the geographical position of each station.

GENERAL PHYSICAL ARTICLES.

THE CALIBRATION AND PERFORMANCE OF THE RAYLEIGH DISC.—E. Barnes and W. West. (*Journ. Inst. Elect. Engineers*, September, 1927, pp. 871-880.)

Owing to recent developments in telephony, the use of the Rayleigh disc for acoustic measurements has become increasingly important. The first part of the paper deals in general with methods of calibration of a Rayleigh disc as an instrument for measuring small air-particle velocities, and describes the method that has been adopted for calibration. The second part is concerned with comparative tests between discs of different sizes and composition at audio frequencies, and with the effects of internal resonance of mica discs. The third part summarises the general considerations affecting acoustic measurements and the errors liable under different conditions of use.

SUR UNE MÉTHODE D'OBSERVATION DE VARIATION DE CONSTANTES DIÉLECTRIQUES (On a method of observing variation of dielectric constants).—G. Guében. (*L'Onde Electrique*, August, 1927, pp. 388-392.)

During the course of a series of investigations on the effect of radioactive radiation on dielectrics, the author was led to seek whether this radiation, which produces an alteration of the conductivity of the dielectric, has also an action on its dielectric constant. To this end a method was studied capable of detecting small variations of capacity resulting from a variation of the dielectric constant. The best method under the given conditions was found to be a modification of that described by E. Meyer in the *Wireless World*, No. 381, p. 805, 1926. The method is explained in detail: its sensitivity is such as to detect a variation of dielectric constant equal to at least .001 of its value. At present the method is being employed to investigate a variation of dielectric constant of a whole series of dielectrics under the action of radium rays. Up to now the tests carried out have yielded negative results.

CHANGEMENTS DES PROPRIÉTÉS OPTIQUES DU QUARTZ SOUS L'INFLUENCE DU CHAMP ÉLECTRIQUE (Changes in the optical properties of quartz under the influence of an electric field).—M. Ny Tsi Ze. (*Comptes Rendus*, 185, pp. 195-197.)

It is found that charging positively the extremity of the electric axis, that would become positive by a compression exerted in the direction of the axis, increases the double refrangibility in this direction and diminishes it in the direction normal to the optical and electrical axes. The phenomena change in sign when the electric field is reversed and their magnitudes are proportional to the fields.

QUELQUES OBSERVATIONS FAITES SUR LE QUARTZ PIÉZO-ÉLECTRIQUE EN RÉSONANCE (Some observations on resonant piezo-electric quartz).—E. P. Tawil. (*Comptes Rendus*, 185, pp. 114-116, 11th July, 1927.)

In a paper last year (*C.R.* 183, p. 1,099) the author described variations in the optical properties of quartz when it vibrates piezo-electrically. The present paper gives a detailed account of further phenomena.

THE TEMPERATURE VARIATION OF THE ELASTICITY OF ROCHELLE SALT.—R. Morgan Davies. (*Nature*, 3rd September, 1927, p. 332.)

Valasek (*Phys. Rev.*, 478, 1922) has studied the temperature variation of the piezo-electric modulus of Rochelle salt. He found abrupt change in the values of this modulus at temperatures of $-15°$C. and 23° C., using crystal slabs with their length at 45° with the b and c crystallographic axes. The object of this note is to point out that there

is evidence for the existence of similar discontinuities in the values of the elastic constants of this crystal at these two temperatures.

A Theory of the Gravitational Field in the Light of Maxwell's Theory.—C. Venkata Row. (*Physical Review*, 30, 2, August, 1927, pp. 189-200.)

The theory starts from the fact that gravitational forces obey the inverse square law in common with electric and magnetic forces; from this it is inferred that Newton's law of gravitation must be developed into a theory of field-action on much the same lines as Coulomb's law has followed; as a first step towards this, the two principles, namely, Hamilton's principle and the special principle of relativity, on which Maxwell's theory is founded, are shown to be sufficient to determine uniquely Lorentz's law of electromagnetic force. Inertia and gravitation are recognised as only different aspects of the same phenomenon. Newton's inertial frame is shown to mark a fundamental physical medium in which matter is embedded, the medium being modified near large masses such as the sun.

Sur les Equations de l'Electromagnetisme (On the equations of electromagnetism).—F. Gonseth and G. Juvet. (*Comptes Rendus*, 185, pp. 341-343.)

A mathematical note with the object of formulating a five-dimensional relativity whose equations supply the laws of the gravitational and electromagnetic fields and the movement of a charged material point also the equation of M. Schrödinger's waves.

Les Equations de l'Electromagnetisme et l'Equation de M. Schrodinger dans l'Univers à Cinq Dimensions (The equations of electromagnetism and M. Schrodinger's equation in the five-dimensional universe).—F. Gonseth and G. Juret. (*Comptes Rendus*, 185, pp. 535-538.)

Light-Quanta and Maxwell's Equations.—N. Rashevsky. (*Phil. Mag.*, 4, 22, September, 1927, pp. 459-465.)

In a recent paper (*Phil. Mag.*, 1926, p. 1,208), Prof. Kasterin attempted to show that, in spite of the generally accepted opinion, the conception of light corpuscles is compatible with Maxwell's equations and that the form of the light-quantum theory, proposed by Sir J. J. Thomson, may be obtained as a particular solution of Maxwell's equations, provided we also consider discontinuous solutions of these equations. The writer here discusses the difficulties involved with a view to elucidating the fundamental question.

Sur la Métrique de l'Espace à 5 Dimensions de l'Electromagnetisme et de la Gravitation (On the metric of the five-dimensional space of electromagnetism and gravitation).—F. Gonseth and G. Juvet. (*Comptes Rendus*, 185, pp. 412-413.)

D. E. H.

Esperanto Section.

Abstracts of the Technical Articles in our last Issue.

Esperanto-Sekcio.

Resumoj de la Teknikaj Artikoloj en nia lasta Numero.

PROPRECOJ DE CIRKVITOJ.

Kelkaj Mezuroj de "Stalloy" (Ŝtalaloja) kerno kun Samtempa Kontinukurenta kaj Alternkurenta Ekscitado.—L. B. Turner.

La artikolo priskribas mezuradojn de "Stalloy"—kerna bobeno, kun grandeco kiel malaltfrekvenca transformatoro, ŝokbobeno, k.t.p. La mezura metodo estas priskribita kaj la rezultoj estas prezentitaj en formo de tabelo kaj serio de kurvoj montrantaj induktancon kontraŭ K.K. miliamperoj por diversaj valoroj de alternanta voltkvanto po 90 cikloj ĉiusekunde. Oni faras rimarkigojn pri la rezultoj kiel helpoj je funkciado kaj desegnado.

La Proprecoj de la Cirkla Diagramo por Telefonaj Frekvencaj Transformatoroj Intervalvaj.—Prof. F. E. Hackett.

La artikolo traktas pri la cirkla diagramo por konsiderado pri la malaltfrekvenca intervalva transformatoro, kiel evoluigita de D-ro. Dyer (en *E.W. & W.E.*, Sep., Okt., & Nov. 1924a). Aparte, ĝi donas alian metodon por kalkulado, kiu estas iom pli simpla, tial, ke ĝi evitas la pezaĉajn esprimojn ordinare uzitajn kiam oni traktas pri paralelaj cirkvitoj. La ekvivalenta cirkvito de la transformatoro estas diskutita kaj esprimo ricevita por la kalkulado de la cirkla diagramo, kies evoluigo estas poste diskutita.

Estas ankaŭ noto pri la utiligo de vektoraj diagramoj rilate al ĉi tiuj kalkuladoj.

RICEVADO.

Altatensiaj Filtrilaj Cirkvitoj por K.K. Elektraj Ĉeftuboj.—J. H. Owen Harries.

La aŭtoro unue diskutas la temon pri ondetado je ĉeftuba provizado, kaj tiam iras al konsiderado pri filtrilaj cirkvitoj. Sekcio traktas pri la determino de praktikaj valoroj de cirkvitaj konstantoj, kaj diagramo estas donita de la fina cirkvito de l'aŭtoro.

Oni ricevas esprimon por la proporcio de elmeta kontraŭ enmeta ondeta voltkvanto kaj por hazardaj kurentoj (aparte ĉe elektraj sistemoj kie la pozitiva ĉeftubo estas terigita). Fina noto traktas la utiligon de filtriloj por kaj Altatensia kaj Malaltatensia provizado.

RADIO-FREKVENCAJ TRANSFORMATOROJ: ILIA APLIKADO AL ŜIRMITAJ VALVOJ.—D-ro. N. W. McLachlan.

La aŭtoro unue konsideras la okazon de radiofrekvenca transformatoro (kun agordita sekundario) kiam uzita kune kun certaj fabrikoj de tri-elektrodaj valvoj difinitaj. La ekvivalenta cirkvito estas analizita kaj esprimo deduktita por pligrandigado. La rezonado estas tiam etendita al okazo de ŝirmita valvo ("Osram" Ŝirmita Valvo S625), kaj la pliboniĝo je amplifado montrita. La aplikado de la ŝirmita valvo al la okazo de la agordita valvo estas poste pritraktita, kaj la aŭtoro finas per sekcio pri la selektiveco de la transformatoro.

LA VALVO KUN ŜIRMITA PLATO KIEL ALTFREKVENCA AMPLIFIKATORO.—R. T. Beatty.

La aŭtoro unue diskutas la proprecojn de valvo kun plene ŝirmita anodo, donante kurvojn por la ŝirmita valvo de Hull. Li poste transiras al komercaj valvoj kun ŝirmitaj platoj, donante karakterizojn kaj detalojn de la inter-elektroda kapacito, kompare kun la okazo de la Valvo Hull'a. Oni priskribas eksperimentojn pri la voltkvanta pligrandigo de krado al plato, kun tabeligo de rezultoj. La aŭtoro tiam traktas pri la stabileco de unuŝtupa amplifikatoro kun agorditaj kradoj kaj agorditaj anodaj cirkvitoj, denove donante eksperimentajn rezultojn obtenitajn per la komerca tipo de ŝirmita valvo.

Fine li diskutas tutan voltkvantan amplifadon obteneblan, lasante tiajn demandojn, kiel selektiveco kaj multŝtupa amplifado, por traktado dum iu estonta okazo.

DIREKTA SENFADENO.

KALKULADO DE LA POLUSAJ KURVOJ DE ETENDITAJ ANTENAJ SISTEMOJ.—E. Green.

La artokolo traktas pri metodoj por kalkuli la proprecojn de direktaj antenoj, kiel ekzemple, la Marconi'a Radia Anteno.

Unue konsiderita estas la okazo de linio da antenoj, ĉiu apartigita per frakcio de ondolongo, kaj la tuto kelkajn ondolongojn longa. La determino de la vektoroj laŭ diversaj direktoj estas donita, polusaj kurvoj estante montritaj por sistemoj 2λ larĝaj kaj 10λ larĝaj. La efekto de reflektilo estas ankaŭ montrita. Poste diskutita estas la energio-pligrandiĝo de etendita antena sistemo, kompare kun unuobla anteno, kaj la energio-pligrandiĝo kaŭze de etendiĝo je la larĝeco de l'antena sistemo. Aliaj sistemoj de etenditaj antenoj estas ankaŭ konsideritaj, inkluzive unu por kuncentrigo de energio en la vertikala ebeno.

ĜENERALAJ FIZIKAJ ARTIKOLOJ.

LA AMPLIFADO DE MALGRANDAJ KURENTOJ PERE DE LA TERMO-RELAJO KAJ LA FOTO-ELEKTRA ĈELO.—J. Taylor.

La temo estas enkondukita per ĝenerala diskutado pri termo-elektraj voltkvantoj kaj termokuploj. Poste estas priskribita la termo-relajo (suldita al Moll), konsistanta el strio de "Constantan Manganin-Constantan" enfermita en vakuigita bulbo. La lumo el spegula galvanometro, reflektita je la centron de la manganino, lasas la sistemon je termo-elektra ekvilibro, sed la defleksigo de la luma punkto detruas la ekvilibron kaj naskas termo-elektran elektromovan forton, kiun oni povas utiligi por funkciigi alian galvanometron, la aranĝo funkciante efektive kiel relajo.

Iom simila aplikado de la fota-elektra ĉelo estas ankaŭ priskribita.

DIVERSAĴOJ.

RESUMOJ KAJ ALUDOJ.

Kompilita de la *Radio Research Board* (Radio-Esplorada Komitato), kaj publikigita laŭ aranĝo kun la Brita Registara Fako de Scienca kaj Industria Esplorado.

MATEMATIKO POR SENFADENAJ KOMENCANTOJ.—F. M. Colebrook. Daŭrigita el antaŭaj numeroj.

La nuna parto traktas pri Vektoroj, Funkcioj kaj la diferencigo de Vektoroj; poste transiras al Integrala Kalkuluso, traktante pri Sendifina Integrado, Integrado de la Sumo aŭ Diferenco de Funkcioj, Integrado per Partoj Definita Integrado, la Meza Valoro de Funkcio, k.t.p.

Correspondence.

Letters of interest to experimenters are always welcome. In publishing such communications the Editors do not necessarily endorse any technical or general statements which they may contain.

The Theory of the Flat Projector.

To the Editor, E.W. & W.E.

SIR,—The article by Dr. J. A. Fleming on "The Theory of the Flat Projector," in the July issue of *E.W. & W.E.*, appears to be misleading in some places.

1. It is asserted that, "owing to the inductance of the reflector wire and its low resistance, the electric current, generated in it, lags nearly $\pi/2$ behind the inducing E.M.F. in phase."

This is not true because the reflector wire, being in length equal to half a wavelength, behaves as a pure resistance. Therefore the current in a reflector wire is in phase with its E.M.F., and as far as this E.M.F. is due to the current in the aerial wire just in front of it, it is $\pi/2$ out of phase with this last-mentioned current (and not π).

2. Further, it is asserted that "this inducing E.M.F. is $\pi/2$ in phase behind the current in the aerial wire." But this is not exact, the E.M.F. being $\pi/2$ in phase *before* the current in the aerial wire (it is understood that the same positive sense is chosen for both wires: aerial and reflector). This can be proved easily, supposing that the inductive field—which has a sinusoidal distribution in space—is stable, and that the reflector wire is moving with the velocity of light towards the aerial wire; then, applying Fleming's right-hand rule, the sense of the E.M.F. can be found (considering only the magnetic induction).

3. In calculating the field due to the current in the first reflector wire at a point P, it is stated that the field is proportional to:

$$sin\left[m\left(\gamma+\frac{\lambda}{4}cos\,\theta\right)-n\left(t-\frac{T}{4}\right)\right]$$

The last term in the brackets

$$n\left(t-\frac{T}{4}\right)$$

is justified by "taking in account the phase difference $T/4$ of the reflector wire current and the E.M.F. creating it."

This would be true—not considering, for the present, the above two points—if the E.M.F. phase is taken as phase origin (in time) for the field. But as it is stated in the article that this E.M.F. lags $\pi/2$ behind the aerial wire current (which creates the inductive field), one must consider a new phase difference $T/4$ in the field expression. Therefore the last term would have been

$$n\left(t-\frac{T}{2}\right)$$

But now considering also the above two points, the reflector wire current being in phase with the E.M.F., the first phase difference $T/4$ does not exist, and then the E.M.F. being $\pi/2$ in phase before the current in the aerial wire, the second difference of phase $T/4$ is of an opposite sign to that considered before.

Therefore the field due to the first reflector wire at a point P is proportional to

$$sin\left[m\left(\gamma+\frac{\lambda}{4}cos\,\theta\right)-n\left(t+\frac{T}{4}\right)\right]$$

when this field has the same phase origin as the field due to the first aerial wire current (in time).

4. Then, if the above points are taken into account we cannot more say that in order to obtain the resultant field we have to subtract algebraically H' from H, but we must add the two fields.

5. The manner in which the numerical example is treated is without interest, because, for different points P of a circle round the emission station, the calculated values of the fields are only instantaneous values for one and *the same moment*. All these values do not represent the maximum values of the fields at the corresponding points, and only a diagram, containing these maximum values plotted against the direction θ, is interesting.

6. In the numerical example given in this article, and also in some places of the general case, it is supposed that the aerial wires are spaced half a wavelength apart ($d=\lambda/2$). However, actually, in the Marconi Beam Stations, $d=\lambda/4$ or $d=3\lambda/4$ is always used. In these cases some of the conclusions to which the author arrives are no more exact and specially it can no more be easily asserted that the current in a reflector wire is due only to the current of the aerial wire in front of it.

London, N.W.1. TUDOR A. TANASESCU.

To the Editor, E.W. & W.E.

SIR,—I do not agree with the criticisms which Mr. Tanasescu makes on my article on the above in your July issue. Nothing is more absolutely certain than that the flat grid aerial of Mr. Franklin casts a wireless shadow behind it. This implies that the field due to the currents in the reflector wires close behind the reflector wires is in opposition with the field at that point due to the currents in the aerial wires.

Hence the reflector wires form a perfect screen as far as backward radiation is concerned and a perfect reflector as regards forward propagation.

The mistake Mr. Tanasescu makes is in ignoring the fact that true radiation does not begin until a quarter wavelength away from the oscillator, and that the magnetic force is propagated initially with an infinite velocity.

At the moment when the aerial current is at its maximum the field due to it at the reflector wire, which is a quarter wavelength away, is also a maximum, and the rate of change of this field and therefore the inducing E.M.F. is at a maximum. Hence even if the section of the reflector wire acts as a pure resistance and its current is in step with the E.M.F. the reflector current must be in opposition as regards phase with the aerial current. The radiation of magnetic force from the aerial wires begins to be propagated a quarter of a

wavelength from them with the velocity of light and the same for the reflector wires fields. Hence, at any distance along the line normal to the plane of the aerial, these forces differ in phase by 90 degrees.

In the direction of the plane of the aerial this difference is 180 degrees.

As we are only considering propagation nearly normal to the aerial, we can approximately take this difference of phase at the same place to be 90 degrees.

I need not deal in detail with the rest of his criticisms as they are mostly beside the mark.

I take the opportunity to correct myself one error which may have confused readers. On page 388 in equation (4) I have taken F to stand for $sin\ (N\pi \frac{d}{\lambda} sin\ \theta)$ and G for $sin\ (\pi \frac{d}{\lambda} sin\ \theta)$. On page 389 I have unfortunately omitted *dashes* over the F and G in the expressions $L+F—G=X$ and $L+F—G+K=Y$. These should be $L+F'—G'=X$ and $L+F'—G'+K=Y$ where $F'=(N\pi \frac{d}{\lambda} sin\ \theta)$ and $G'=(\pi \frac{d}{\lambda} sin\ \theta)$.

The title of my article is an "approximate theory," and it makes no pretence to being an exhaustive one. All I desired was to bring out the fact that the directive and beam effect of this form of aerial essentially depends upon *interference*. The results of the simple theory are however in general agreement with the facts as stated by Senatore Marconi and also as I have heard them privately from the Engineers of the Marconi Company.

J. A. FLEMING.

The Audio-Transformer Problem.

To the Editor, E.W. & W.E.

SIR,—I wish to thank Mr. P. K. Turner for his further letter in your September issue and am sorry that, owing to an error of my newsagent by which that issue was delivered to me a fortnight late, I am delayed a month in offering him apologies for my unkindness.

Mr. Turner's statement that it *is* the case for him that "L_1 will be as large as we can make it" brings us, I feel, to the root of the matter. We all want satisfactorily to reproduce frequencies down to say 25 cycles, so I take his statement to mean that on a commercially practicable manufacturing basis the core size and winding spaces of the transformer will be so reduced as to result in the required primary impedance at 25 cycles being the maximum mechanically possible within the limits of fine winding.

I would like, however, to ask Mr. Turner this question: "Supposing he were designing such a transformer without particular regard to cost, but with particular regard to good performance and with a moderate regard to size and weight, would it not then be the case that such a first-class transformer was such that its primary winding inductance was not the highest possible in the space occupied by that winding? Would he not use a rather larger core section and winding space, thereby obtaining the same inductance with a considerably reduced primary ohmic resistance? Could not self and mutual capacity effects be somewhat reduced by the greater freedom possible in the design of the windings?" In this connection it is to be noted that the reduction in the working flux density would compensate for the increased volume of iron in such a manner that hysteresis and eddy current losses would not be increased. In addition the H effect of the steady current would be reduced and this in itself would have the (secondary) effect of increasing the available inductance as well as the available amplitude of low frequency operation. Low frequency operation would also be assisted by the reduction of primary ohmic resistance.

Consider the effect of

A 2 per cent. increase in all the linear dimensions of the iron and winding spaces.

This would give for the same inductance

A 1 per cent. decrease in the number of turns.
" 3 " " " flux density.
" 4 " " " primary resistance.
" 3 " " " steady H.

No change in eddy current loss.

No appreciable increase in hysteresis loss.

The secondary effect of higher inductance due to smaller steady H.

There is the one snag, viz., that, with similarly formed windings, the capacities would be increased 2 per cent. This is of no consequence on the primary. (Technically, therefore, the argument of larger size applies with great force to an *output* transformer, whose primary impedance *on open secondary circuit* must be several times that of the valve even at 25 cycles.) My suggestion is that the added liberty of design, resulting from the increased dimensions, would enable us largely to avoid the increases in the secondary and mutual capacities, and therefore to produce a transformer equally good on high and better on low frequencies and one for which L_1 is not as large as we can make it. It would if we keep the same number of secondary turns, also have a higher ratio and amplification.

I have no connection with any concern manufacturing audio transformers and no opportunity of making any but simple tests on these: my interest in them is purely scientific, and as such it seems to me that no firm has yet produced the really first-class article. Also that when it is produced, it will be such that L_1 is not as large as mechanically possible. I call to mind a make of audio transformer of which I believe this is true, which make I regard as the nearest approach to the ideal.

I shall await with interest further communications from your manufacturing correspondents showing me, if such is the case, that the production of a still better transformer on the lines suggested is impracticable on some other basis than the question of cost.

There is one other point arising out of Mr. Turner's letter, and that is that the equality principle as between effective primary impedance and valve impedance is still applicable, on the assumption of a perfect coupling, in the case of a loaded secondary of a fixed number of turns, when the primary is varied. In the stage case the load is considered to be a capacity in series with the secondary resistance, and in the output transformer case, a fixed speaker load connected to the secondary.

Derby. E. FOWLER CLARK.

To the Editor, E.W. & W.E.

SIR,—Now that the problem of the A.F. transformer primary has been settled, might we not consider the *secondary*?

Some few makers put out a "1st Stage" transformer of rather high ratio—some 5 : 1 or 6 : 1, with another for "2nd Stage" of about 2 : 1. On close examination we find that these have identical primary windings, the idea of leaving out some of the secondary being to avoid overloading the output valve. Quite eminent manufacturers have adopted this system, and, of course, many little people have copied them without any clear idea on the subject. The practice does not appear to be quite sound: it is a dodge rather than a solution—alas, all too common in wireless!

So far as I can recollect, there was little science and no logic in the design of transformers available to the public up to about four years ago, when a very old-established firm in this country brought out a pair of transformers to suit the best valves then available; it was evident that much thought had been given to the problem of the primary windings, and the ratios were such as to give good strength with plenty in hand, whatever the considerations were that led to the choice of the values. Other firms—some of whom should have known better—floundered along for years by rule of thumb, or no rule at all. Now that there is more general understanding of what a transformer should do, most makers seem to choose a primary winding to suit a given class of valve, and then fill up the rest of the space with secondary, subject to considerations of self-capacity, etc., but at least one firm keeps the same secondary winding through a whole series of instruments, varying the number of primary turns to suit the valve after which the transformer is to be used.

At first sight, one is inclined to agree with the school that "fills up the rest with secondary," subject to the obvious precautions, but are there not other factors which might influence design? The opinions of people qualified to hold them should be of great interest.

Plympton, Devon. L. J. VOSS.

Amplification of Small Currents by Means of the Thermo-Relay.

To the Editor, E.W. & W.E.

SIR,—With reference to the article entitled "The Amplification of Small Currents by means of the Thermo-Relay, etc." in your October issue, will you kindly allow me to point out that the method described was evolved by me prior to 1919 and is fully described in my Patent Specification No. 144757 of 1919, and also in a paper read by Miss T. D. Epps and myself before the Physical Society of London. Vol. xxxii., Part V., p. 326, August, 1920.

At the time this method aroused little interest, and it is satisfactory to see that it is now receiving the attention it merits; but I think it unfortunate that no acknowledgment of our work should be made.

All that Messrs. Moll and Burger appear to have done is to substitute their own particular form of thermocouple for those described in my patent.

Surbiton, Surrey. W. H. WILSON.

[From the following reprint from the *Proceedings of the Physical Society of London* to which reference is made in the letter, it will be seen that Mr. Wilson's priority in the matter is beyond question. It is unfortunate that no acknowledgment or reference to this earlier work was made by Dr. Taylor, but we presume that it was unknown to him. We were present when Mr. Wilson's paper was read in 1920 and subsequently used his method of making thermo-junctions for use in thermo-ammeters, but yet we had so entirely forgotten that he had suggested their use for magnifying galvanometer deflections, that we regarded Moll and Burger's suggestion as something quite new. We are pleased to know that the idea originated much nearer home than Utrecht.

From the *Proceedings of the London Physical Society*, Vol. xxxii., p. 338, 1920.

"Another useful arrangement consists of two lines of junctions connected in opposition and arranged close together as shown in Fig. 15. If radiant heat be arranged to fall in a line 1 mm. in width symmetrically about the axis *A-B*, the thermo-E.M.F.s generated in the two halves will be equal and opposite, but a movement of the heat line of ½ mm. to either side will cause it to cover entirely one set of junctions or the other, resulting in a deflection of the galvanometer from one side of zero to the other side of zero. Since the number of junctions in each line may be made large by these methods, the deflection of the galvanometer *G* may be made substantially proportional to the movement of the band of radiant heat.

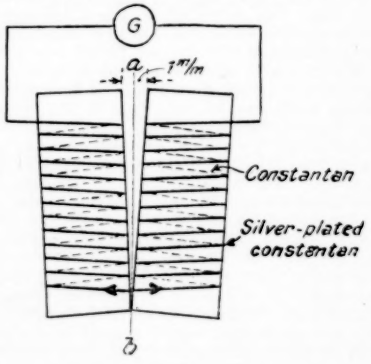


FIG. 15.

"The arrangement, therefore, comprises a useful means of magnifying small movements. The band of radiant heat may be the image formed by the mirror of another reflecting galvanometer, hence it is possible to cause a deflection produced by this galvanometer to produce a considerably larger deflection in the galvanometer *G* connected in the thermo-electric circuit."—ED., *E.W. & W.E.*]

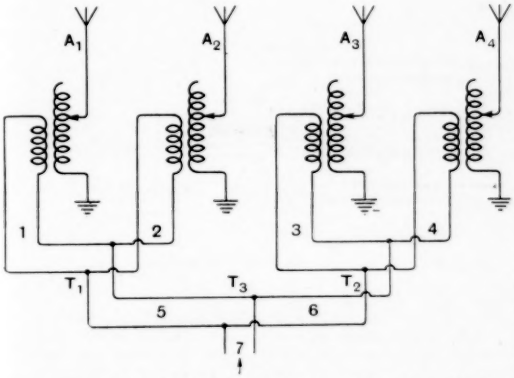
Some Recent Patents.

The following abstracts are prepared, with the permission of the Controller of H.M. Stationery Office, from Specifications obtainable at the Patent Office, 25, Southampton Buildings, London, W.C.2, price 1/- each

AERIAL FEEDERS.

(*Application date, 30th April,* 1926. *No.* 274,970.)

Extended aerial systems A - - - A_4, such as are used for directive "beam" transmission, are fed with high frequency currents at a number of points along their length as shown. In this connection difficulties arise owing to reflection effects occurring at the various junction points T_1, T_2, T_3 of the feeding cable, and the consequent creation of stationary waves. For instance, in any section of feeder cable in which the terminal load is not equal to the surge impedance of the line, stationary waves will be set up, but at points in the cable one-quarter wavelength apart, voltage and current will be in phase, and the net effect of the load and cable is that of a pure resistance.



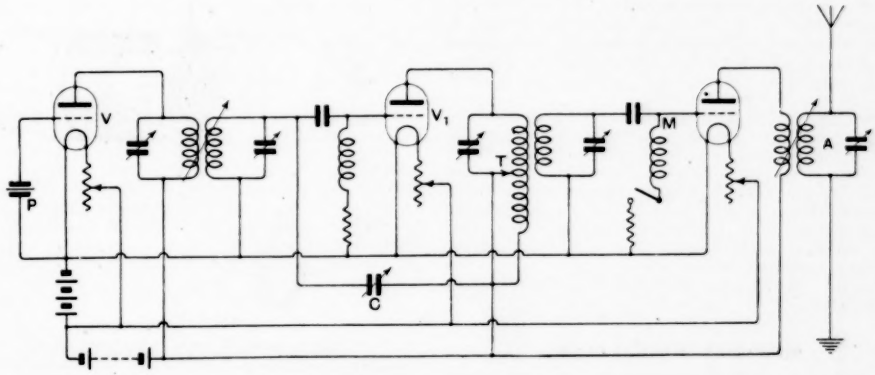
Advantage is taken of this fact to arrange the effective resistance to equal the surge impedance and thus avoid all reflection effects. The aerial loads are first adjusted so that there are no stationary waves set up in the Sections 1—4. The Sections 1 and 2 will however constitute a load upon the Section 5, equal to that thrown by the Sections 3 and 4 upon the Section 6. The invention consists in making the Sections 5 and 6 each an odd number of quarter wavelengths long, so that the effective load upon the junction T_3 becomes a pure resistance which can be made equal to the surge impedance of the line 7.

Patent issued to E. Green.

FREQUENCY STABILISERS.

(*Application date, 18th August,* 1926. *No.* 274,660.)

In the ordinary method of coupling a crystal-controlled oscillator to successive stages of amplification, a temporary breakdown of the crystal oscillator usually results in the transmitting gear being placed out of action. In order to overcome this difficulty the master-control valve V, containing a piezo-crystal P in its grid circuit, is coupled to a valve V_1, which is itself capable of generating sustained oscillations through the interaction of tuned plate and grid circuits. By a back-coupling from the point T through a condenser C, the valve V_1 is neutralised to such an extent that so long as the crystal P is in operation, only those oscillations of predetermined frequency are passed through to



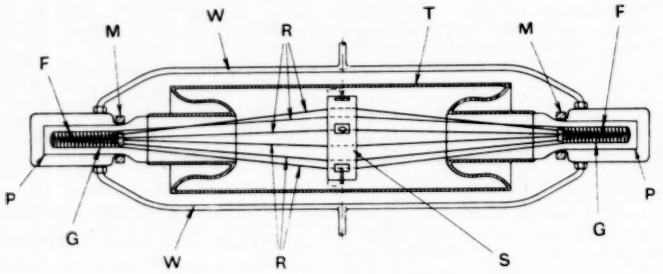
the modulator M and aerial A. Should the crystal break down, however, the neutralising means can be readjusted to the point where the valve V_1 itself generates oscillations sufficient to maintain transmission, independently of the frequency control stage V.

Patent issued to C. W. Goyder.

SHORT-WAVE GENERATOR.

(*Application date, 13th April, 1926. No. 274,183.*)

The multi-electrode tube illustrated has been designed to generate high frequency oscillations varying from ten million up to several hundred million cycles per second. In order to obviate the enormous capacity leakages occurring at the higher



ranges, the whole of the circuit connections, including a part of the radiating system, is enclosed within the same evacuated vessel as the tube electrodes.

The two co-acting sets of electrodes P, F, G are mounted at the opposite ends of a sealed tube T, and are connected in pairs by a series of straight rods R. The anodes are water-cooled, the cooling liquid circulating through pipes W. A central steatite member S, mounted inside the vessel T, forms a support for the rods R, and also constitutes the point at which the filament voltage is supplied to the system. The anode voltage is applied through the water pipes W.

The generated high frequency oscillations are located partly in the internal rods R joining the two grids, and partly in the external water tubes W connecting the plates. Radiation may take place directly from the tube system, but the latter is preferably coupled inductively to a separate tuned radiator. Guard rings M protect the glass seal of the containing vessel from the effects of high frequency electric stress.

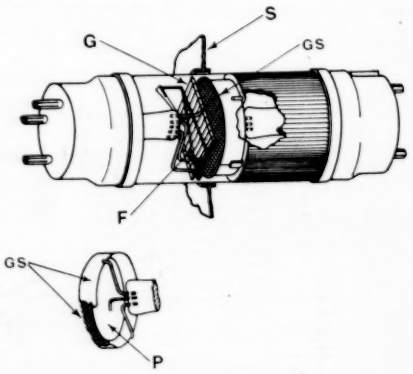
Patent issued to W. J. Brown and Metropolitan Vickers Co.

SHIELDED GRID VALVE.

(*Application date, 5th May, 1926. No. 275,335.*)

In order to eliminate the effect of inter-electrode capacity between the plate and grid circuits of an amplifying valve, an additional grid is interposed between the ordinary control grid and the plate, and is designed to shield one from the electrostatic field of the other. As shown in Fig. 1, all four electrodes are arranged transversely to the glass tube, *i.e.*, one behind the other, as distinct from the ordinary arrangement in which the grid and filament are mounted inside a cylindrical plate.

The filament F is V-shaped and is mounted inside a flattened spiral winding G forming the ordinary control grid. Both these electrodes are supported from a glass foot at one end of the containing vessel. The shielding grid $G\,S$ consists of a disc of metal gauze fixed to a metal rim, the whole forming a bowl-shaped member (shown separately in the figure). The plate P consists of a plain disc of metal lying inside the periphery of the grid $G\,S$. These two electrodes are preferably mounted in a glass foot located at the other end of the containing vessel to that supporting the filament and control grid. Corresponding contact pins are provided at each end of the bulb as shown. In order to increase the shielding action of the grid $G\,S$, an auxiliary



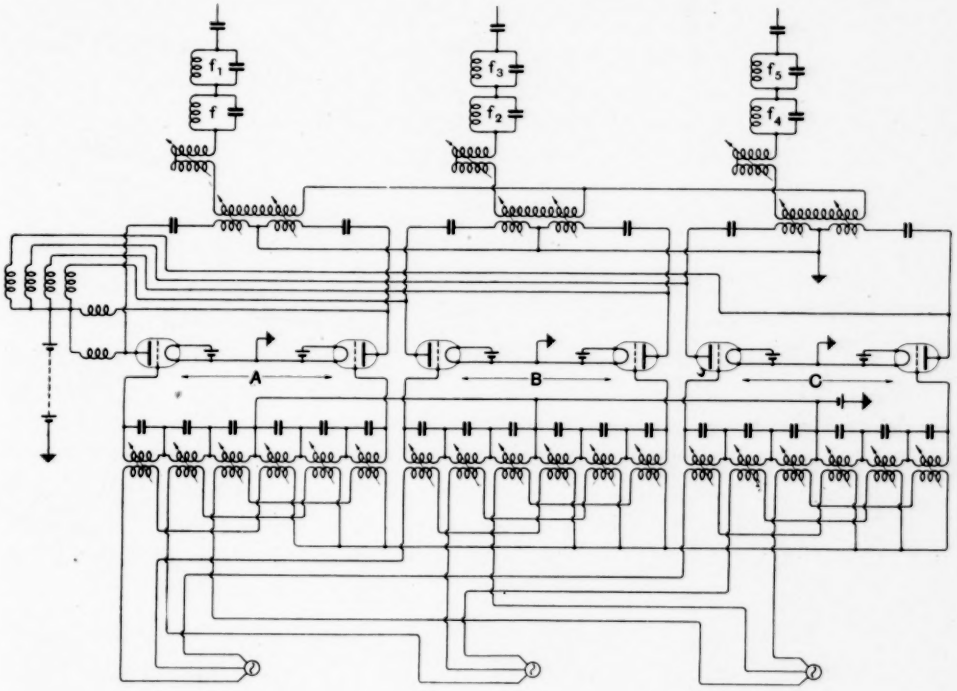
shielding member S, terminating in a flattened rim, is mounted outside the bulb and close to the grid $G\,S$. In operation this external member is directly earthed, whilst the shielding grid is given a biasing voltage of 80, assuming an operating voltage of 120 on the plate.

Patent issued to H. J. Round.

"POLYPHASE" WIRED WIRELESS.

(*Convention date (U.S.A.), 23rd December, 1925. No. 263,777.*)

Wired Radio Inc. propose to use triphase carrier current, each phase of which is separately modulated, for distributing alternative broadcast programmes simultaneously by wired wireless. The mains of a polyphase power-transmission system from the distributing network.



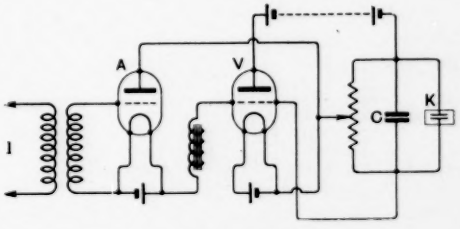
As shown in the figure, modulated carrier frequencies f_1, f_2, f_3 are fed from triphase generators, 1, 2, 3 to the input circuits of three pairs of balanced H.F. amplifiers *A*, *B*, *C*, the coupling-coils being arranged as shown in order to suppress the even harmonics. Other harmonics are eliminated by filter circuits f_1 . . . f_5 in the outgoing network.

PHOTO-ELECTRIC APPARATUS.

(*Convention date (Germany), 18th August, 1925. No. 257,268.*)

The so-called Kerr effect relates to the rotation of plane polarised light when reflected from a magnetised surface. A similar rotation occurs when passing a ray of light through a cell containing carbon bisulphide placed in an electrostatic field. It is obvious that such an arrangement may find a useful application in television and similar apparatus where the conversion of electric currents into corresponding optical effects is involved.

According to the present invention the normal sensitivity of a Kerr cell is increased by subjecting it to the influence of high frequency oscillations, in addition to the low frequency signalling impulses. As shown in the figure a Kerr cell *K* is bridged



across a condenser *C* in the plate circuit of a back-coupled valve *V*, generating oscillations of the order of 10^6 cycles per second. Low frequency impulses,

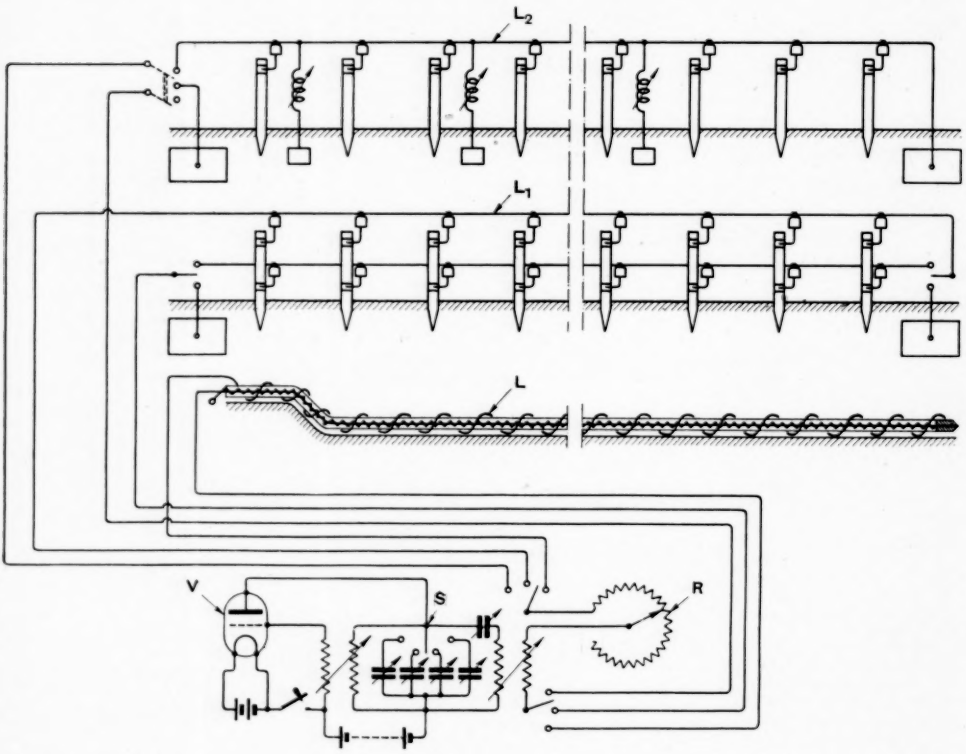
corresponding to the "picture elements" or other signals are applied at *I* through an amplifier *A*, inserted across the grid and filament of the generator *V*.

Patent issued to the Telefunken Co.

CABLE LEADER GEAR.

Convention date (France), 15th April, 1925. No. 250,933.)

The navigation of tortuous channels by ships, or the landing of aeroplanes at night or in foggy weather, can be assisted by means of a system of operation at will by means of a switch *S*. An automatically-rotating rheostat control arm *R* can be used to vary the intensity of the currents fed into the "leader" cables, thus giving the navigator an opportunity of gauging his actual distance from, as well as his bearings relatively to, the guide cables. If, for instance, the current supply is maintained at constant strength, a constant range of reception from any given point results. When however the amplitude of the supply current is varied, the duration of the individual signal notes detected in the receiver will increase the nearer the observer approaches to the energised cable. In this way



cables fed with low frequency current. The cables are laid along the course to be followed, and their location is detected by picking-up the spreading inductive fields due to the low frequency currents. For this purpose a valve amplifying set is carried by the vessel under navigation.

According to the present invention currents of different frequencies can be fed, either simultaneously or successively, into a submarine line *L*, or into overhead "leader" cables L_1, L_2 from a single oscillating valve *V*, comprising a number of differently-tuned circuits, which can be brought into a shortening of the silent period between successive notes indicates that the vessel is closing-in towards the energised cable.

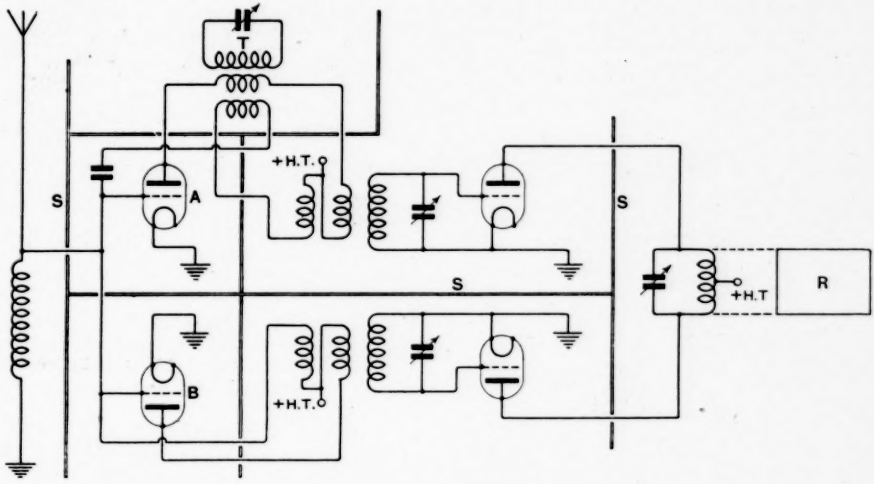
Patent issued to Société Industrielle des Procédés —W. A. Loth.

INTERFERENCE ELIMINATORS.

(Application date, 30th October, 1926. No. 276,195.)

The invention is based upon the known method of preventing interference in which two separate circuits are fed from the same aerial in such a way

that one circuit transfers the interference effects only, whilst the other circuit transfers both the desired signal and the interference. By opposing the outputs from the two circuits in a common receiver, the interference effects are cancelled out, leaving the desired signal alone to reach the detector. The inventor points out that the inter-electrode capacity of the relay valves used in such a system is a disturbing factor, and claims the use of suitable neutralising and screening means to prevent interaction between the circuits.



As shown in the figure two separate branches *A* and *B* are provided between the aerial and the common receiver *R*, the valves being suitably balanced for capacity coupling and shielded by metal screens *S*. The branch *B* transfers both the desired signals and interference. A wave trap *T*, tuned to the desired signal frequency, is coupled to the branch *A*, and absorbs the desired signal component from that circuit. The result of combining the two outputs is to cancel out the common component of undesired frequency. The residue is the desired signal and appears alone in the detector *R*.

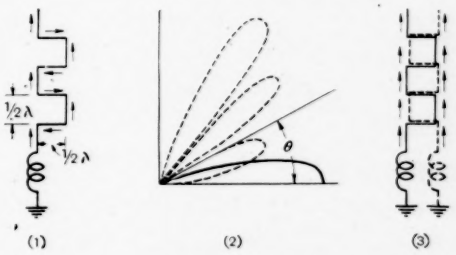
Patent issued to R. Custerson.

SHORT-WAVE AERIALS.

(*Application date*, 10*th March*, 1926. *No.* 267,540.)

The antennæ system comprises wires or layers bent at right angles in sections corresponding to half wavelengths as shown in Fig. 1. To an observer located on the horizon, in a line perpendicular to the plane of the aerial, the vertical current components indicated by the arrows are additive in phase, since they are all at approximately the same distance from him, so that the vertically polarised electric field will be at a maximum. Should the observer now move upwards, say in a balloon, still keeping perpendicular to the plane of the aerial, the vertically polarised field will be found to be distributed as shown in the shaded polar diagram of Fig. 2, whilst horizontally polarised fields will be discovered having the distribution shown in dotted lines in Fig. 2.

If the observer moves out of a line perpendicular to the plane of the aerial the distribution of the vertical and horizontal fields will greatly change, owing to the increase in effective distance between successive elements of the bent aerial. In all cases, however, such an aerial system is characterised



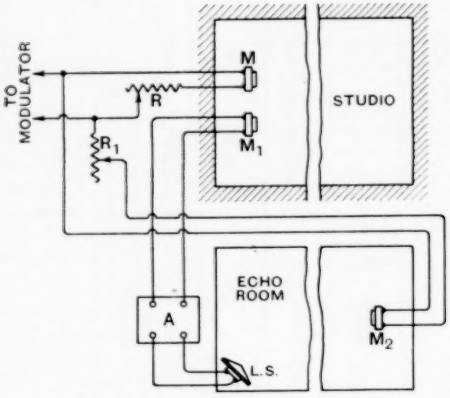
by the radiation of energy in which the polarisation of the electric field changes suddenly according to the position of the receiver. Fig. 3 represents a combination of two bent aerials in which the effect of the horizontal segments is neutralised. The principle is also applied to aerials bent in three dimensions.

Patent issued to Société Française Radio-Electrique.

REVERBERATED BROADCAST.

(*Application date, 13th May, 1926. No. 276,052.*)

In order to subdue the "first echo" effect to pleasing proportions, it is usual to drape the walls of the broadcasting studio. This, however, robs the transmission of the subtle quality of the more prolonged echoes or reverberations, and produces a somewhat "dead" tonality at the receiving end. The object of the present invention is to overcome this defect by restoring the missing reverberations, which are produced separately in an undraped room at a distance from the studio proper.



As shown in the figure two microphones marked M, M_1 are provided in the studio. The first feeds the modulator circuit directly, whilst the second is connected through an amplifier A to a loud-speaker located in the "echo" room. In the same room is a third microphone M_2 so situated as to receive the reverberations set up in that room, whilst avoiding as far as possible any direct pick-up from the loud-speaker. The current from the microphone M_2 is fed into the modulator circuit in parallel with that from the main instrument M, the relative strength of the direct and "reverberation" components being adjusted by means of rheostats R, R_1.

Patent issued to H. J. Round.

GRID-CONTROL MODULATION.

(*Application date, 8th July, 1926. No. 275,771.*)

The output from a power valve O is modulated by varying the conductivity of an auxiliary valve V, which is inserted in the grid-filament circuit of the former and functions as a grid-leak. Modulating current from the microphone M is applied across the plate and grid of the valve V. A high resistance $G\,L$ of the order of 90,000 ohms is inserted in series with the secondary winding of the microphone transformer, and serves to regulate the grid potential of the valve V, so that its internal impedance is automatically controlled throughout the whole range of the applied modulating voltage. In an alternative arrangement the resistance $G\,L$ is omitted, and a third valve is shunted across the

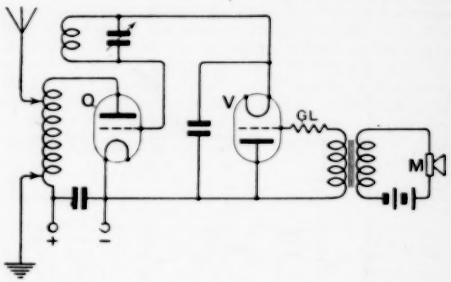


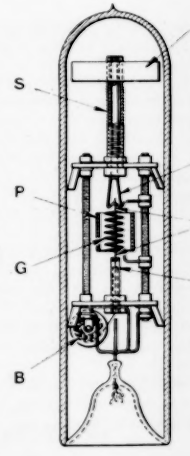
plate and grid electrodes of the valve V, the microphone voltage then being applied across the grid and filament of the added valve.

Patent issued to N. F. S. Hecht and G. Morton.

RENEWABLE VALVE FILAMENTS.

(*Convention date (France), 22nd June, 1926. No. 273,293.*)

A supply of tungsten wire is mounted on a bobbin B inside the casing of the valve, and means are provided for replacing a burnt-out section of filament without breaking the seal. The active portion of filament is held between two clips C, C_1, the upper of which can be made to move downwards into contact with the lower by rotating a soft-iron member M along a screwed spindle S under the influence of an external magnet.



When the filament burns out, the upper clip C is accordingly brought down through the spiral grid G and plate P until it reaches the conical nose of the clip C_1. Further movement causes the clip first to open and release the old end of wire, and then to take a fresh grip on the remaining piece of tungsten projecting from the clip C_1. On its backward movement a new length of filament is drawn from the bobbin B, the pressure of the clip C_1 on the wire being less than that of the clip C in order to allow the wire to slip through.

Patent issued to C. Tourne.